Order Number 9401034

A model of asset restructuring announcements

Yancey, William Frederick, Ph.D.

The University of Texas at Austin, 1993

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

A MODEL OF ASSET RESTRUCTURING ANNOUNCEMENTS

APPROVED BY
DISSERTATION COMMITTEE:

J. Reed Smith

D. Paul Newman

Senyo Y. Tse

Dale O. Stahl, II

Richard D. MacMinn

Dedicated to my wife, Carol, whose love and patience were vital to its completion.

A MODEL OF ASSET RESTRUCTURING ANNOUNCEMENTS

by

WILLIAM FREDERICK YANCEY, A. B., M. F., B. A., M. B. T.

DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT AUSTIN

August 1993

ACKNOWLEDGMENTS

I benefited from the insights of many people during the writing of this dissertation. I appreciated the comments and patience of my committee members: Reed Smith (chairman), Paul Newman, Senyo Tse, Dale Stahl, and Richard MacMinn. Jaewan Park provided many useful insights. Participants at workshops at Texas Christian University, University of Tulsa, and Syracuse University also made helpful comments.

I acknowledge financial support from the American Institute of Certified Public Accountants Grants-In-Aid for Doctoral Dissertations in Accounting. Mathematica® software was provided by the Neeley School of Business at Texas Christian University.

A MODEL OF ASSET RESTRUCTURING ANNOUNCEMENTS

Publication No. ____________

William Frederick Yancey, Ph. D.

The University of Texas at Austin, 1993

Supervisor: J. Reed Smith

To explain stock market reaction to asset restructuring announcements, this dissertation constructs and analyzes an asymmetric information model with costly signalling. Firm managers privately observe the firm's future cash flow opportunities from a continuous distribution, and make a public announcement at the beginning of one operating period. If the announcement is false, at the end of the operating period the firm pays a penalty under SEC Rule 10b-5 that depends on the magnitude of lying. The stock market reacts rationally to announcements considering the firm's incentives. An exogenous compensation contract motivates the manager to tradeoff the benefit of lying versus the lying penalty .

The analysis in Chapter 4 identifies Bayesian Nash equilibria of the model for various settings. When the firm privately observes its productivity type, there does not exist an equilibrium that fully reveals the type and allocation decision to the investors. When the penalty rate is sufficiently low, a pure pooling equilibrium exists in which all types mimic the highest type. When the penalty rate is sufficiently high, a partial pooling equilibrium exists in which the range of types is partitioned into a series of intervals, each with its own unique announcement. The equilibria are sensitive to changes in the exogenous parameters.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

In the past several years, many publicly-traded corporations in the United States and other countries have announced an unprecedented number of restructurings of operating assets. In a restructuring announcement firms announce how they plan to terminate or expand various operating business segments. Recent empirical research has found conflicting evidence on whether the capital markets react favorably or unfavorably to these announcements. Studies that aggregate all restructuring announcements find the mean market reaction is slightly negative [Elliott and Shaw 1988; Fried et al. 1989; Strong and Meyer 1987]. However, studies focusing on specific types of restructuring announcements find a significant positive market reaction to reorganization of operations [Lindahl and Ricks 1991], divestiture [Klein 1986], liquidation [Kim and Schatzberg 1985], and project termination [Statman and Sepe 1989].

The restructuring announcements of Xerox in 1989 and 1993 are a case study in how the capital market may react positively or negatively. On January 31, 1989, Xerox announced a $275-million write-down of businesses in document-processing workstations and other areas and a return to focus on its core businesses in high-performance photocopiers and financial services [*Wall Street Journal*, February 1, 1989, p. C19; Norman 1989]. Over the next three trading days in 1989, the price of Xerox common stock *increased* by 3.6 percent. On January 18, 1993, Xerox announced a $778-million write-down of insurance and financial service businesses and another return to focus on photocopiers and printers [*Wall Street Journal*, January 19, 1993, p. A3]. Over the next three trading days in 1993, the price of Xerox common stock *decreased* by 4.0 percent.

Attempting to explain the capital market reaction to restructuring announcements leads to two inter-related questions. (1) How do investors interpret some restructuring announcements differently than others? (2) What motivates a firm manager to announce a restructuring, if he anticipates investors will interpret that announcement as "bad news"?

Understanding the market reaction to restructuring announcements is important to financial analysts and accounting regulators. As expected sales growth in many industries is revised downwards and firms announce many operating changes, analysts can rely less on extrapolation of past trends to predict cash flow.

Recently, regulators have been concerned about whether existing standards are adequate for disclosures about operating changes. In 1989, the Securities and Exchange Commission issued Financial Reporting Release 36 that specifically encouraged firms to improve management's estimates of future performance in the annual Management Discussion and Analysis [Heyman 1989; Hooks and Moon 1991]. For several years the Financial Accounting Standards Board (FASB) has been struggling with a new standard for disclosure of asset writedowns including assets intended for disposal during a restructuring [Bodner and Kiss 1988; Chang and Nichols 1991; Clark and Lorensen 1987; Schuetze 1987]. The FASB issued a

Discussion Memorandum on asset writedowns in 1990 and plans to issue an Exposure Draft with a proposed new standard in 1993.

This dissertation develops a costly signalling model to demonstrate how restructuring announcements can reveal information that causes stock market price changes. Prior analytical accounting research has developed single-period models with costly signalling to explain how a firm's private information about future cash flows could be revealed by voluntary direct disclosure [e. g., Feltham and Xie 1992; Verrecchia 1983], auditor selection [e. g., Bachar 1989; Datar et al 1991], or accounting method choice [Jung 1989]. In many of these prior models, operating policy is fixed and the firm's only strategic decision is reporting strategy. This dissertation considers a more complex environment where a firm can move operating assets among several business segments and chooses both an operating and a reporting strategy.

At the beginning of the operating period in my model the firm manager privately observes the cash flow productivity of each business segment and makes two concurrent decisions: a private operating decision to allocate resources among segments, and a reporting decision on the direction and magnitude of changes to publicly announce to investors. I assume the firm makes its announcement in the form of a press release or in the annual management discussion and analysis (MD&A) filed with the Securities and Exchange Commission (SEC). Although the firm is required to make an announcement, I allow the firm to lie by choosing an announcement that differs from its actual allocation. At the end of the operating cycle, audited financial statements reveal the actual allocation. Investors can compare the firm's announcement to the actual allocation, and force firms to pay a penalty under SEC Rule 10b-5 that depends on the magnitude of lying.

The firm manager in my model is motivated by an exogenous compensation contract to maximize a linear combination of the firm's stock price at the beginning and end of the operating cycle. The stock price at the beginning of the operating cycle depends on investors who observe the firm's announcement and revise their beliefs about the firm's end-of-period value. Investors adjust their valuation response for the expected amount of lying. The stock price at the end of the operating cycle depends on the actual allocation of resources to each division and the amount of the Rule 10b-5 penalty, if any.

Managers who observe that their expected cash flow will be below average have an incentive not to reveal their type. If they announce a restructuring that fully reveals their low type, then the stock price will be low both immediately after the announcement and at the end of the period when cash flow is realized. If the investors believe there is some correlation between firm value and announcements, then low type firms have some incentive to lie and mimic the announcements of the high type firms. The compensation-maximizing announcement choice is a tradeoff between the benefits of lying related to the increase in stock value immediately after the announcement and the cost of lying related to the penalty at the end of the period when the amount of lying is revealed.

In my analysis I show how the firm's equilibrium announcement and allocation strategy change in response to changes in the parameters. If the penalty rate for lying is sufficiently low, then all types lie. If the exogenous penalty rate for lying is sufficiently large, then there exists a partial pooling equilibrium such that the range of feasible types is partitioned into a large number of intervals, each with a distinct announcement.

My approach differs from most voluntary disclosure models in accounting, finance, and economics. In most voluntary disclosure models, firms have a dichotomous choice between nondisclosure and full truthful disclosure. If the firm chooses to disclose, then those models assume an instantaneous verification mechanism that would inflict immediate penalties on any false announcement. An instantaneous verification mechanism is difficult to justify for announcement of prospective resource allocations, since the comparison of the announcement to the actual allocation cannot be made until the end of the operating period. In my model firms can make false announcements, but they incur a penalty when they do.

The partial pooling equilibrium in this dissertation has some similarities to the partition equilibria in Crawford and Sobel [1982] and Newman and Sansing [1993]. These models have equilibria in which a continuous range of feasible types is partitioned into intervals, and each interval makes a different announcement. In Crawford-Sobel and Newman-Sansing the announcements are "cheap talk," and each firm type is indifferent over any announcement in a specified interval. In my model the Rule 10b-5 penalty makes the announcements costly, and firms are not indifferent among announcements. Therefore, the partial pooling equilibrium in my model is characterized by a unique announcement for each interval.

The remainder of this dissertation is structured as follows. The second chapter is a literature review of the assumptions in signalling models. The third chapter sets up the model and justifies assumptions. The fourth chapter analyzes equilibria in a series of propositions. The fifth chapter discusses the results and limitations of the model. The Appendix contains proofs of the propositions.

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

This chapter places my model of restructuring announcements in the context of other models of financial disclosure in the accounting, finance, and economics literature. In the first section of this chapter I discuss how restructuring announcements differ from the disclosures analyzed in previous models. The subsequent sections of this chapter discuss the essential assumptions of previous models and identify aspects of those models that I have incorporated into my model.

I define a restructuring announcement as an announcement by a firm of its intentions to move resources among operating divisions. The details of my model set-up are presented in chapter 3, and are summarized as follows. I assume the firm privately observes some information about the future productivity of each division, privately makes an action choice on how to allocate future resources, and decides what to announce to investors about its future resource allocation. The future cash flow of the firm depends on the productivity and amount of resources at each division. Investors observe the restructuring announcement, infer the firm's private information (productivity of each division) and the firm's private action (resource allocation), and estimate future cash flow.

Earnings or cash flow announcements have been the focus of most prior discretionary disclosure models in accounting [for example, Darrough and

Stoughton 1990, Dye 1985a, Feltham and Xie 1992, Verrecchia 1983 and 1990; Wagenhofer 1990]. In discretionary disclosure models, the firm privately observes information on earnings or cash flow, chooses truthful disclosure or nondisclosure, but makes no operating decision. Those models have been used to show how much information is voluntarily disclosed when firms are not subject to mandatory disclosure regulation. I examine announcements in the more complex environment of restructurings where the firm chooses both an operating and disclosure policy.

Each of the following sections discusses one of the following categories of assumptions:

2.2. Manager's private information

2.3. Announcements observed

2.4. Investors' response

2.5. Incentives

2.6. Solution concept

Within each section I compare assumptions made in prior models and describe the assumption I make in my model. Another comprehensive review of disclosure models is available in Xie [1991, chapter 2].

2.2. Private Information

In financial disclosure models some or all firms privately observe some firm-specific information that is relevant to firm valuation. This private information is described as the firm's *type*. The probability distribution of types is common knowledge, but the realization is not.

Different disclosure models have different definitions of firm type. Most of the disclosure models in accounting and finance define type as the firm's liquidating value [for example, Newman and Sansing 1993, Verrecchia 1983] or current economic earnings [for example, Miller and Rock 1985], and assume the firm's only decision is to choose a disclosure policy. Other models allow the firm manager to make an operating decision on allocating productive resources, and define type as a productivity parameter [Lanen and Verrecchia 1987; Trueman 1986, 1990]. In these latter models the firm's terminal value is a function of the productivity parameter and the operating decision. My model adopts the latter assumption.

Most disclosure models assume all firm managers observe their own type with certainty before making an announcement. My model follows the common practice and assumes all firm managers know their type. A few models assume some managers do not observe their own type [Dye 1985a, Section 3; Jung and Kwon, 1988]; and that implies if a firm does not disclose, investors do not know if firms are ignorant or deliberately trying to hide their information.

2.3. Announcements observed

The literature includes models where firms may communicate their private information by direct disclosure (direct disclosure models) or costly actions (signalling models). Depending on the set up assumptions and the parameter values, the models have equilibria in which announcements may fully, partially, or never reveal the firm's type.

Most direct disclosure models restrict firms to two choices: fully revealing disclosure or nondisclosure [Darrough and Stoughton 1990, Dontoh 1989, Dye 1985a, Feltham and Xie 1992, Grossman and Hart 1980, Jovanovic 1982, Verrecchia 1983 and 1990; Wagenhofer 1990]. These models assume firms would receive a severe "death" penalty if they chose disclosure and that disclosure did not fully reveal the type. Any false or partially revealing announcements would be detected by a costless nonstrategic verification mechanism, such as a nonstrategic external auditor.

In my model I assume that the restructuring announcement cannot be immediately verified, because the auditor has to wait until the end of a future operating cycle to determine if resources were actually allocated as the firm announced. I assume that firms making false announcements are discovered at the end of the operating cycle and pay a penalty that is an increasing function of the amount of lying.

Discretionary disclosure models allow firms to choose nondisclosure, because they assume disclosure is not required by mandatory reporting requirements. However, the SEC specifically requires firms to announce resource allocation plans (see section 3.1 for institutional details). Therefore, my model does not allow nondisclosure and requires all firms to announce how future resource allocations compare to the current allocation. Firms can announce the resource allocation will remain at current levels, be shifted slightly, or subject to a major restructuring. In my model firms choose between truthful and false disclosure; whereas in discretionary disclosure models, the choice is between truthful and no disclosure.

When firms are restricted to choosing either full disclosure or nondisclosure, the nature of disclosure costs determines whether nondisclosure is an equilibrium strategy. Grossman [1981] and Milgrom [1981] showed that if there are no costs associated with full disclosure, then all firms must fully disclose, because any firm choosing nondisclosure is assumed to be the worst type.

Verrecchia [1983] assumed any firm that fully discloses incurs the same positive constant exogenous disclosure cost. Disclosure cost determines the threshold for disclosure. Firms with type greater than the threshold choose full disclosure, because the benefits of disclosure exceed the cost. Low type firms choose nondisclosure. When investors observe nondisclosure, they know the firm is in the nondisclosure interval, but they do not know the exact type. In Verrecchia as disclosure cost increases, the threshold increases; more firms are below the threshold; and less disclosure occurs.

Darrough and Stoughton [1990] develop a model of an incumbent firm in an industry and assume disclosure cost increases with firm type. If high type firms disclose, then potential competitors will enter the industry and profits of the incumbent high type firm will be reduced. If low type firms disclose, then potential entrants will not enter the industry. In Darrough-Stoughton, as the cost of entry rises, entry by competitors is less likely, and there is more disclosure.

Both Verrecchia and Darrough-Stoughton have a partial disclosure equilibrium, but the effect of a change in disclosure cost is different. In Verrecchia, the high type firms truthfully disclose and increasing disclosure cost *decreases* disclosure. In Darrough-Stoughton, the low types fully disclose, and increasing entry cost *increases* disclosure.

Wagenhofer [1990] extends Darrough-Stoughton to continuous types. In Wagenhofer there is a partial-disclosure equilibrium with two distinct nondisclosure intervals. One interval of very low types does not disclose to avoid the adverse reaction of stock market investors. Another intermediate interval of types does not disclose to avoid the adverse reaction of potential entrants in the product market. The investors' equilibrium response to nondisclosure is an expectation over the nondisclosure intervals. Two intervals of full disclosure may exist, a high interval where the benefits of full disclosure exceed the cost of entry by the competitor, and an intermediate level where the firm type is sufficiently low that the competitor does not enter when type is disclosed. In Wagenhofer increasing entry cost may either increase or decrease disclosure, depending on parameter ranges.

Dye [1986] assumes the firm's private information has two components: nonproprietary information x that may reduce cash flows by a small amount if disclosed, and proprietary information y that will significantly reduce cash flows if disclosed. Dye allows firms to choose between full disclosure (both x and y), partial disclosure (x but not y), and nondisclosure (neither x nor y). An exogenous statistical relation between x and y exists, so that if only x is disclosed, the investors can revise their expectations of y. Dye finds a range of parameter values such that firms with low x and low y choose nondisclosure; firms with high x and high y choose full disclosure; and firms with some intermediate types choose partial disclosure. The sensitivity of disclosure equilibria to changes in the costs of disclosure depends on both the absolute costs of disclosure and the relative costs of disclosing nonproprietary x and proprietary y.

In the set-up assumptions of both Dye [1986] and my model, the firm has private information about two value-relevant dimensions. In Dye's model the two dimensions are x and y, and both are random states chosen by Nature. The two dimensions in my model are the productivity parameter θ, chosen by Nature, and the resource allocation a, a private action chosen by the firm. Dye assumes an exogenous statistical relation between the two dimensions, whereas I derive equilibrium results in chapter 4 that show an endogenous relation between θ and a.

In my model the investors cannot precisely predict firm cash flow unless they can infer the correct value of both θ and a. Firms are allowed to make false announcements $(a \neq \hat{a})$, but incur a penalty that increases with the magnitude of lying $(a - \hat{a})$. In my model's equilibria the highest type firm makes an announcement $\hat{a}$ that truthfully discloses its allocation a, but this announcement is mimicked by lower type firms. I show that increasing the penalty for false announcements $(a \neq \hat{a})$ induces all types to make more truthful reports of their allocation a. However, no matter how high the penalty for false announcements, the firm's equilibrium announcement strategy will only partially reveal the type θ.

Thus, the equilibrium announcement in my model is a noisy revelation of the firm's type-allocation pair $\{\theta, a\}$.

Noisy revelations are a feature of Crawford and Sobel [1982] and Newman and Sansing [1993], which assume, as I do, that disclosures are not instantaneously verified. These models have partition equilibria where the feasible types are partitioned into intervals, and each interval has a different announcement. When the announcement is made the investors can infer the interval that contains the firm's type, but only the firm manager knows the exact value of type. The precision of the investors' expectation increases as the interval length decreases. In all three models the number and size of the intervals in equilibrium are sensitive to changes in the exogenous parameter values.

In Crawford-Sobel there is a difference between the preferences of the Sender (firm) and Receiver (investors). The difference is specified by an exogenous parameter, b. The value of b is common knowledge. As b approaches zero, the difference in preferences is eliminated, the Sender prefers to fully disclose information, and the number of information partitions is infinite. When b is sufficiently large, the difference in preferences is so large that the Sender prefers nondisclosure. Nondisclosure in Crawford-Sobel is a babbling equilibrium where all Sender types randomize over the entire feasible message space. The intuition from the Crawford-Sobel result is that information disclosure is more informative when the Sender-Receiver preferences are more closely aligned.

In Newman-Sansing the firm's announcements are observed by two uninformed players: a stockholder and a potential competitor. The firm's incentives are perfectly aligned with the shareholder. The payoffs of the firm and stockholder may be reduced if the potential competitor enters the market. For some parameter settings (Newman-Sansing Propositions 3 and 4), the competitor's entry decision is not likely to depend on the firm's announcement; and the firm prefers to make a more informative disclosure to the stockholder. For other parameter settings (Newman-Sansing Corollary 1), the competitor's decision is sensitive to the firm's announcement, and the firm makes noisier disclosures to deter entry. Even though the stockholder generally prefers more precise information, the stockholder receives some benefit from noisy signals that deter entry by the competitor. If the loss in firm revenue from competitive entry is low (Newman-Sansing Propositions 5 and 6), then high type firms make more precise disclosures and entry occurs, while low types make less precise disclosure and entry is deterred. More precise disclosure by the high types offsets the stockholder's loss from the competitor's entry. Thus, there are parameter settings in Newman-Sansing where the high type firms announce relatively high-precision "good news;" and the low type firms announce relatively low-precision "bad news."

The partial pooling equilibrium in section 4.5 of this dissertation are sensitive to exogenous parameters: the lying penalty rate $\hat{k}$ and the compensation function parameters, β_1 and β_2. Unlike Newman-Sansing the firm's incentives in my model are not perfectly aligned in my model. One result of my model is that as $\hat{k}$ decreases or β_1 increases, the low types are more likely to mimic a higher type. The highest type announces his high type, but that "good news" announcement is mimicked by some lower types. One characteristic of my partial pooling equilibrium is that the high intervals are relatively longer than the lower intervals. For the lowest types, the benefits of mimicking are so small that the low types make announcements with relatively little noise.

The relationship between "good news" and precision in my model's partial pooling equilibrium result differs from the Newman-Sansing result. In my model "good news" announcements are *less precise* than the "bad news." In Newman-Sansing "good news" is *more precise* than "bad news." The differing assumptions about firm-investor conflicts drives these differing results. In my partial pooling equilibrium the incentives are not well-aligned, and the lowest types are prevented from lying by a sufficiently costly exogenous penalty for lying. In Newman-Sansing the incentives are perfectly aligned, and some noise by the lower types benefits the stockholder by deterring entry by a potential competitor.

In signalling models firms publicly announce costly actions to convince investors of the firm's future value. In a signalling model the announced actions may be an adoption of a method for financial or tax accounting, dividend policy, issue of stock or bonds, auditor selection, or operating decision. For example, when a firm privately observes that future sales and inventories will increase, it may signal its type by adopting the LIFO method of inventory accounting [Jung 1989]. Some models allow firms to use both direct disclosures *and* publicly observable signalling actions, such as equipment replacement decisions [Lanen and Verrecchia 1987], market entry [Farrell 1987], regulatory compliance action [Kambhu 1988], verification by investment bankers [P. Hughes 1986], or stock ownership retained by the entrepreneur [Datar, Feltham, and J. Hughes 1991].

The essential characteristic of signalling models is that the incremental cost of signalling must be related to firm type. Separating signals occur when the signalling cost is sufficiently high that: (i) the low type prefers to be revealed as low rather than incur the cost of signalling, and (ii) the high type prefers to incur the signalling cost and be revealed as high. Pooling occurs when the signalling cost is either: (i) so low that the low types mimic the signal of the high types, or (ii) so high that the high types avoid the costly signal and are indistinguishable from low types.

My model is a costly signalling model because the firm's announcement has a direct effect on the firm's cash flow and the payoff to the firm manager. In my model the highest type maximizes its payoff by announcing a major restructuring in a particular direction. In the pooling and partial pooling equilibria in chapter 4, lower types incur a penalty from mimicking the announcement of higher types, but the amount of the penalty is less than the benefit from deceiving the investors.

2.4. Investors' response

In all financial disclosure models there are investors who observe the firm's announcement to estimate the firm's value and make some response action. The investors' responses differ significantly across the models. In some models there are always current shareholders at any point in time who satisfy personal liquidity preferences by selling their stock at expected value in a competitive stock market [Dye 1986, Miller and Rock 1985, Teoh and Hwang 1991]. My model adopts this assumption. Other disclosure models focus on public offerings and assume that after the announcement investors buy most of the firm [Allen and Faulhaber 1989, Wagenhofer 1990], or contribute additional equity for expansion [Darrough and Stoughton 1990, Feltham and Xie 1992]. An alternative assumption is that shareholders are endowed with the firm's stock and use their estimate of firm value to make a decision regarding how much to consume currently [Newman and Sansing 1993].

The investors in most financial disclosure models are "reaction machines" that respond based on their expectations of the firm's value. Unlike the owners in agency contracting models, investors in disclosure models are not allowed to make binding commitments before the firm makes its announcement.

Most disclosure models in the accounting literature focus on public disclosures by firms and simplify the capital markets by assuming that all investors are identical. They assume all investors have the same prior beliefs and evaluate new information at the same time and in the same way. I assume the restructuring announcements in this model are public and interpreted by investors in the same way. Other models, such as Grossman [1976] and Verrecchia [1982], focus on how private information obtained by individual traders influences market behavior and assume investors have diverse prior beliefs that are updated by observing stock prices in the capital market.

2.5. Incentives

All financial disclosure models assume a conflict such that some firms prefer not to fully reveal all private information to the investors. Without such a conflict, the investors could implement an incentive contract that motivates the manager to fully reveal [Dye 1985a, section 5; Myerson 1979] The following paragraphs discuss different approaches where the conflict may arise from an adverse third party, differing objectives between manager and shareholders, or differing time preferences among shareholders.

Models with adverse third parties describe the firm's type as proprietary information that an adversary could use to take actions that decrease the firm's future cash flow. Possible adversaries are current or potential competitors in product markets, tax collectors, regulatory agencies, creditors, consumer advocates, labor unions, or disgruntled employees. Some models with proprietary information are constructed with two representative players, either a firm and an investor [Verrecchia 1983] or a firm and an adversary [Dye 1985b, Kambhu 1988]. Other models explicitly model all three players: firm, investor, and adversary [Darrough and Stoughton 1990, Feltham and Xie 1992, Newman and Sansing 1993, Wagenhofer 1990]. These models assume away moral hazard problems and perfectly align the incentives of the firm manager and shareholders. The manager has conflicting objectives between disclosing all private information to the investors and withholding some information from the adversary. Since private disclosures to investors are not allowed, there are some conditions where the optimal policy for the firm and shareholders is nondisclosure or noisy disclosure.

Some disclosure models assume that there are conflicting objectives between owners and shareholders that cannot be resolved by a full revelation contract. Crawford and Sobel [1982] assume the preferences for the firm (Sender) and the investors (Receiver) differ by some exogenous parameter, *b*. Dye [1985a, section 4] constructs a model where the manager and investors have different private sources of noisy information about the firm's value, and shows that both parties prefer that the manager's information not be announced until after the market has responded to the investors' information. Dontoh [1989] assumes that investors do not know if a firm manager is a current-value or future-value maximizer. Sansing [1990, chapter 3] assumes that investors do not know if a firm manager is a profit-maximizer or an investment-maximizer.

A third group of disclosure models assumes shareholders have conflicting consumption preferences across time [Dye 1988, Miller and Rock 1985, Teoh and Hwang 1991]. These models assume that at any point in time there are some shareholders who wish to liquidate their stock and consume or adjust their portfolio mix. The incentive functions are a linear combination of firm value at two points of time: time 1, immediately after the announcement is observed, and time 2, when the end-of-period cash flows are revealed. The relative weights on the two points of time are determined exogenously. My model follows the third assumption and assumes that the manager's incentive function is a linear combination of firm's market value at the two points of time.

2.6. Solution Concept

The solution concept for most disclosure models is the *Bayesian Nash equilibrium.* This solution concept is called the Bayesian Nash equilibrium by some authors [Crawford and Sobel 1982, p. 1433; Newman and Sansing 1993], and the perfect Bayesian equilibrium by others [Green and Laffont 1990, p. 256; Matthews et al. 1991, p. 251; Rasmusen 1989, p. 110; Teoh and Hwang 1991, p. 286].

The following summary of this solution concept is adapted from the Sender-Receiver game in Crawford and Sobel [1982, pp. 1433-1434] to disclosure games with two players, labeled firms and investors. This solution concept guarantees that in equilibrium the investors extract all available information from the firm's disclosures.

A Nash equilibrium is a combination of strategies such that each player takes the action that maximizes his or her own expected utility taking the action of other players as given. The original Nash concept applied only to games of complete information. Harsanyi [1968] extended this solution concept to games of incomplete information by developing the Bayesian Nash equilibrium. Players without private information, such as investors or adversaries in disclosure models, calculate expected utility with respect to their probabilisitic beliefs about the other player's type.

Sequential rationality implies the players' actions maximize utility given their beliefs after observing the other player's actions and that these beliefs are consistent with the other players' strategy [Matthews et al. 1991, p. 250].

If the firms make an in-equilibrium announcement, then investors update beliefs using the standard Bayesian revision formula described below.

$$f(\theta|\hat{a}) = \frac{g(\hat{a}|\theta)\, f(\theta)}{\int_{\theta\in\Omega} g(\hat{a}|\theta)\, f(\theta)d\theta} \quad \text{for } \theta \in \Omega \text{ and } \hat{a} \in A^e$$

where θ is the firm's type;

Ω is the set of feasible types;

$\hat{a}$ is the firm's publicly observed announcement;

A^e is the set of equilibrium announcements;

$f(\theta)$ is the prior probability density function of firm type;

$f(\theta|\hat{a})$ is the posterior probability density function; and,

$g(\hat{a}|\theta)$ is the likelihood that a manager of a type θ firm will announce $\hat{a}$.

In equilibrium, the investors' beliefs about the firm's announcement strategy, $g(\hat{a}|\theta)$, must be consistent with the strategy chosen by the firm manager. The Bayesian Nash equilibrium concept requires that the players' "... conditional probabilistic beliefs about each other's actions and characteristics are self-confirming" [Crawford-Sobel, p. 1433]. In the context of this model if the investors' beliefs were not consistent with the firm manager's strategy, then the investors' pricing response would be biased, and expected profits would not be zero.

Consider what would happen if the investors' beliefs were not consistent with the firm manager's strategy. Suppose all firms with $\theta \in [0.5,1]$ announced $\hat{a} = 1$, but the investors believed that the announcement $\hat{a} = 1$ was made only by a manager of a type $\theta = 1$ firm. The investors' price response to the $\hat{a} = 1$ would be based on the expectation that type was $\theta = 1$, but the actual firm value be based on a distribution of $\theta \in [0.5,1]$. Thus, the investors with these incorrect beliefs would, on average, overvalue firms that announce $\hat{a} = 1$, and would incur losses if they bought or sold the firm's stock. Investor pricing strategies that incur losses on average cannot be sustained in a Bayesian Nash equilibrium. The investors could improve their response and achieve zero expected profits in this example by responding to the in-equilibrium announcement $\hat{a} = 1$ with beliefs that are consistent with the firm's strategy.

Suppose there is a pure pooling equilibrium where all firms make the same announcement, $\hat{a}^P$. Using the above notation, the firm's strategy is $g(\hat{a}^P|\theta) = 1$ for

all $\theta \in \Omega$ and $f(\theta|\hat{a}^P) = \dfrac{g(\hat{a}^P|\theta)\, f(\theta)}{\int_{\theta\in\Omega} g(\hat{a}^P|\theta)\, f(\theta) d\theta} = \dfrac{f(\theta)}{\int_{\theta\in\Omega} f(\theta) d\theta} = f(\theta)$ for $\theta \in \Omega$.

This implies the investors do not update their beliefs after observing $\hat{a}^P$.

Investor beliefs must be exogenously specified for out-of-equilibrium announcements. For example, suppose $\hat{a}^O$ is a feasible announcement, $\hat{a}^O \in A$, but $\hat{a}^O$ is not observed in equilibrium. This implies $g(\hat{a}^O|\theta) = 0$ for all $\theta \in \Omega$; and the Bayesian revision formula fails because the denominator is zero. The investors' response to an out-of-equilibrium announcement depends on their out-of-equilibrium beliefs about the firm's type, $f(\theta|\hat{a}^o)$. If the modeler did not specify out-of-equilibrium beliefs, the investors' response to an out-of-equilibrium announcement would be undefined. Proving that a firm will not deviate to an out-of-equilibrium announcement is not possible unless the investors' response is defined for all feasible announcements. The out-of-equilibrium beliefs for my model are discussed in section 3.7 and section 4.5.3.

Direct disclosure models that restrict firms to a dichotomous choice between full disclosure and nondisclosure (cited in section 2.3) have simple out-of-equilibrium beliefs. For example, suppose the feasible types are $x \in [0,1]$ and announcements are denoted $\hat{x}$. If all firms with type $x \in [0.5, 1]$ fully revealed and all $x \in [0, 0.5)$ chose nondisclosure, then the set of out-of-equilibrium announcements is $\hat{x} \in [0, 0.5)$. Given the restriction that all disclosures must be truthful, if an out-of-equilibrium announcement, $\hat{x} \in [0, 0.5)$, is observed, then the investors believe the firm's type is $x = \hat{x}$. If all types $x \in [0,1]$ fully reveal the truth, then all announcements $\hat{x} \in [0,1]$ will be observed in equilibrium. "Nondisclosure" would be an out-of-equilibrium announcement. If the equilibrium is full truthful revelation, then most modelers assume that if the firm deviated to an out-of-equilibrium nondisclosure that the investors believe the firm is the lowest type, x=0.

Some disclosure models have partially revealing equilibria where all feasible announcements are observed in equilibrium. Crawford and Sobel [1982] and Newman and Sansing [1993] have noisy partition equilibria where the sets of feasible announcements and types are partitioned into intervals. Each type is assigned to a continuous interval of types and a continuous interval of announcements. The firm's equilibrium announcement strategy is to randomize over all announcements in that interval. Figure 1a illustrates the announcement strategy in a partition equilibrium with three noisy intervals. For example, type $\theta = 0.8$ would randomize over any announcement in the interval $[0.67, 1]$. If Figure 1a represents an equilibrium announcement strategy, then all announcements in $[0, 1]$ are observed in equilibrium and there are *no* out-of-equilibrium announcements.

Figure 1. Comparison of observed announcements.

Figure 1a. Announcements observed in an equilibrium with three noisy intervals.

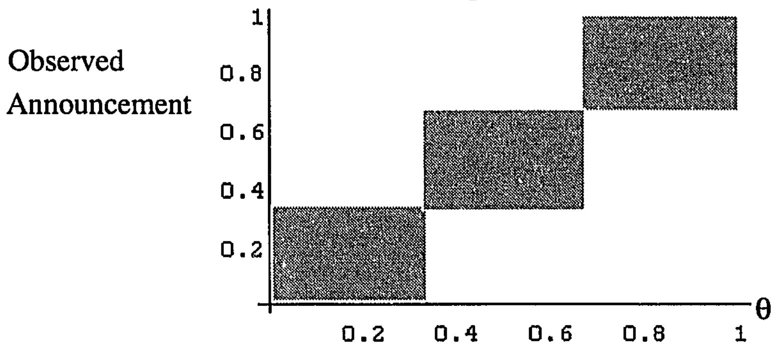
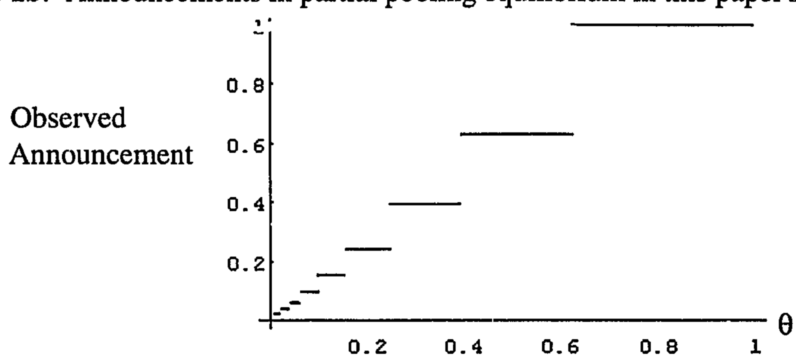


Figure 1b. Announcements in partial pooling equilibrium in this paper's chapter 4.



Out-of-equilibrium beliefs are an issue in my paper because I develop some equilibria where some announcements are not observed in equilibrium. Figure 1b illustrates the announcement strategy in the partial pooling equilibrium developed in chapter 4 of this paper. The feasible set of types is partitioned into many intervals, and each interval is characterized by a unique discrete announcement. Each type does not randomize over an announcement interval. For example, type θ $=0.8$ strictly prefers to announce $\hat{a} = 1$. If Figure 1b represents the in-equilibrium announcement strategy, then there are many out-of-equilibrium announcements. For example, all announcements strictly between 0.63 and 1 would not be observed in equilibrium. Thus, I need to exogenously specify some investor beliefs for these out-of-equilibrium announcements. My assumptions about the investors' out-of-equilibrium beliefs are discussed in section 3.7 and section 4.5.3.

CHAPTER 3

MODEL SET-UP

This chapter consists of seven sections:

3.1. Setting and sequence of events

3.2. Manager's private information

3.3. Firm's valuation function at time 2

3.4. Manager's compensation function

3.5. Investors' valuation function at time 1

3.6. Equilibrium

3.7. Investors' out-of-equilibrium beliefs

3.1. Setting and sequence of events

The financial reporting context for this model is announcements by firms to investors about prospective restructuring of a firm's operating assets. The institutional requirement for announcing future operating plans is based on the Management Discussion and Analysis (MD&A) disclosure rules of the Security and Exchange Commission (SEC). Firms can be penalized for false announcements by litigation under SEC Rule 10b-5.

Since 1974 the SEC has required firms to include a MD&A in their annual 10-K filings to help investors estimate a firm's future performance [Fedders and Perry 1984]. In 1989, the SEC issued Financial Reporting Release 36 (FRR 36) to interpret the MD&A requirements [Heyman 1989, Hooks and Moon 1991]. FRR 36 states,

> ... Required disclosure is based on currently known trends, events, and uncertainties that are reasonably expected to have material effects, such as: A reduction in the registrant's product prices; erosion in the registrant's market share; changes in insurance coverage; or the likely non-renewal of a material contract. ...
>
> A disclosure duty exists where a trend, demand, commitment, event, or uncertainty is both presently known to management and reasonably likely to have material effects on the registrant's financial condition or results of operations. ...
>
> Disclosure of planned material expenditures is also required, for example, when such expenditures are necessary to support a new, publicly announced product or line of business. ...
>
> In preparing MD&A disclosure, registrants should be guided by the general purpose of the MD&A requirements: to give investors an opportunity to look at the registrant through the eyes of management by providing a historical and prospective analysis of the registrant's financial condition and results of operations, with particular emphasis on the registrant's prospects for the future [SEC 1989].

In addition to the MD&A disclosures in the annual 10-K, firms may announce prospective resource allocations in other SEC filings, press releases, and financial analyst meetings.

A recent example of a restructuring announcement by TriCare, Inc., appeared in *The Wall Street Journal*, [May 3, 1993, p. C19]. TriCare operated two major businesses, medical evaluation and medical treatment. On April 30, 1993, TriCare announced it would discontinue its medical evaluation business and layoff almost half its employees. The firm also announced it would incur a $6 million dollar restructuring charge against earnings, and realize about $10 million from disposing of assets and receivables that would be used toward expanding the remaining medical treatment business. TriCare's stock price dropped by 21 percent in the next three trading days.

The firm's announcements about its future resource allocations influence investors' expectations of the firm's future cash flow. To maximize the current stock price, managers may deliberately announce plans that they know the firm will not be able to achieve. Audits by certified public accountants give an opinion on whether the firm's financial statements fairly represent the historical results of operations, but do not give an opinion on the accuracy of the firm's plans for future operations. At the end of an operating cycle, investors can compare the audited results to the plans announced by the management. If there is a significant discrepancy, then the investors can allege the management deliberately made false announcements to deceive investors.

Firms can be sued for false disclosure under SEC Rule 10b-5 by either the investors or the SEC. The legal and empirical research on shareholder litigation related to Rule 10b-5 is reviewed in a recent empirical accounting research paper by Skinner [1992, section 2]. The rule was issued by the SEC in 1942 as authorized by Section 10(b) of the 1934 Securities Exchange Act. The operative words of Rule 10b-5 are:

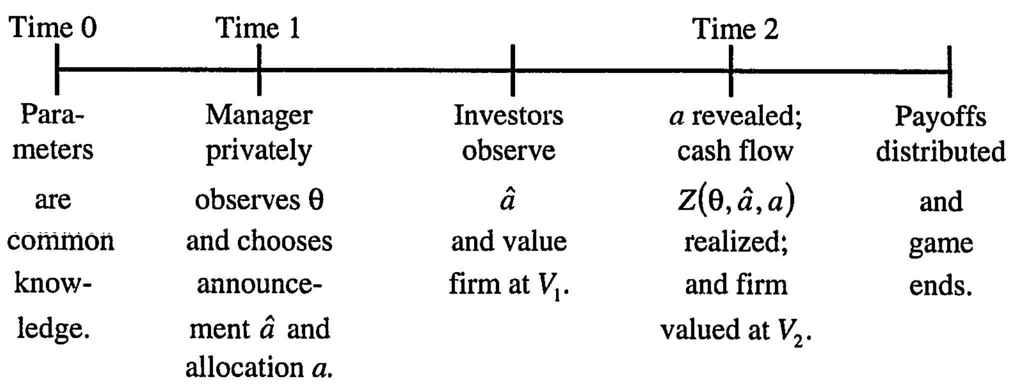
> It shall be unlawful for any person ... to make any untrue statement of a material fact or omit to state a material fact necessary in order to make the statements made, in the light of the circumstances under which they were made, not misleading, ... [17 *Code of Federal Regulations* section 240.10b-5].

Rule 10b-5 has been the basis for a large amount of litigation by shareholders claiming firms have not adequately disclosed material facts that are relevant to firm valuation [Jacobs 1980, Loss 1983]. If a publicly-held firm discloses significant bad news, there are plaintiffs' lawyers eager to file lawsuits in federal court alleging violations of Rule 10b-5 and other securities regulations - often within hours of the bad news disclosure [Moses 1992, O'Brien 1991].

Most investors' claims under Rule 10b-5 are settled before a trial verdict [Alexander 1991, pp. 524-526]. The settlement amounts in these shareholder class-action lawsuits often run into the millions of dollars. For example, Tenneco paid fifty million dollars to settle a shareholder lawsuit alleging misrepresentation [*Wall Street Journal*, April 22, 1992, p. B4]. Although the SEC has not specified a formula for settlement amounts in Rule 10b-5 cases, the courts have developed have established that the amount of settlement is an increasing function of the decline in stock price from the time when the firm made its false statement to the time when the truth is revealed [Alexander, pp. 515-517]. If the amount of loss is relatively small, litigation is unlikely [Alexander, pp. 511-513].

From these institutional features, this chapter constructs a simplified model of a firm making financial announcements subject to a possible penalty for false announcements. The principal features of the model are introduced in this section and discussed in more detail in the following sections. The model has two players, a firm manager and a homogenous group of investors. The time horizon is a single operating cycle as illustrated by the timeline in Figure 2.

Figure 2. Timeline.



The firm has two operating divisions, labeled ESTABLISHED and DEVELOPMENT, that operate in different industries. ESTABLISHED produces goods or services in a stable mature industry where the productivity parameters are common knowledge to all firms and investors. DEVELOPMENT produces goods or services in a rapidly changing industry where past productivity is not necessarily a good predictor of future performance. At time 1 the manager privately observes θ, the productivity parameter of the DEVELOPMENT division. This realization, θ, is drawn by Nature and referred to as the firm's *type*. Section 3.2 describes the type in greater detail.

The firm is endowed with a fixed amount of scarce resources, such as working capital, skilled labor, or competent administrators, that must be allocated among the two divisions. The total amount of scarce resources is normalized to one. After observing θ the manager allocates the resources to the divisions, a to DEVELOPMENT and 1-a to ESTABLISHED. Since the focus of this research is on restructuring of operations rather than financing, firms are not allowed to acquire additional resources by issuing more debt or equity capital. Thus, the manager's choice of a is restricted to the continuous interval [0, 1].

The model assumes the resources are initially allocated equally to each division. As explained in section 3.2 the model assumes that at time 0 there is a fifty percent chance that the firm's productivity θ in DEVELOPMENT is better than ESTABLISHED, and fifty percent chance that it is not. Therefore, it is reasonable that at time 0 resources should be equally allocated to the divisions.

Alternatively, the initial allocation of resources could be set to a_0, an unrestricted exogenous parameter. However, adding another exogenous parameter increases the complexity of the notation. The analysis of the model could be extended to consider the effect of changes in the initial allocation, but that extension is not likely to produce significantly different results.

The difference between the allocation a chosen by the firm and the initial allocation of one-half indicates the direction that resources are moved. If a=1, then ESTABLISHED is shutdown and all resources are transferred to DEVELOPMENT. If a=0, then DEVELOPMENT is shutdown and all resources are transferred to ESTABLISHED. If a is one-half, then resources remain equally divided between the operating divisions.

Concurrent with the allocation decision a, the manager makes a nonbinding publicly observable announcement, $\hat{a}$, of the resources to be invested in DEVELOPMENT. The feasible space of announcement $\hat{a}$ is the continuous interval [0, 1]. Since the motivation of this model is to study the how the magnitude and direction of the restructuring announcements influence investors' expectations, I do not complicate the model by allowing the firm manager to strategically choose the timing of the announcement. Announcement timing has been a primary issue in other financial disclosure models, such as Trueman [1990].

This model allows the firm manager to use a strategy of misleading investors by choosing an announcement $\hat{a}$ that differs from its allocation a. Both $\hat{a}$ and a are selected from the continuous interval [0, 1]. At time 2 the allocation a is publicly revealed, and the firm will be penalized if a is not equal to $\hat{a}$. This penalty for false announcements is described below in section 3.3. The manager's compensation function discussed in section 3.4 creates incentives for some managers such that the benefits of making a false announcement at time 1 are less than the penalty at time 2.

V_0, $V_1(\hat{a})$, and $V_2(\theta, \hat{a}, a, V_1)$ are the valuations of the firm at times 0, 1, and 2, respectively. V_0 is the investors' expectation of the firm's residual value before any announcement is observed. $V_1(\hat{a})$ is the investors' expectation immediately after the firm's announcement $\hat{a}$, and is described in section 3.5. $V_2(\theta, \hat{a}, a, V_1)$ is the residual cash value of the firm distributed to investors at time 2 and is described in section 3.3. The residual value is the cash remaining after the firm pays any penalty for lying and compensation to the manager. The manager's compensation $W(\theta, \hat{a}, a, V_1)$ depends on $V_1(\hat{a})$ and $V_2(\theta, \hat{a}, a, V_1)$ as described in section 3.4. V_1 is an argument in $V_2(\theta, \hat{a}, a, V_1)$ because an increase in $V_1(\hat{a})$ increases the compensation paid to the manager and that decreases the residual cash to the investors. Section 3.3 describes $V_2(\theta, \hat{a}, a, V_1)$ in detail.

My model has uncertainty in only two places. At time 0 both the firm manager and investors are uncertain about the type θ that will be drawn by Nature. At time 1 the manager knows the realized value of θ and his action choice a; but the investors do not directly observe θ or a. The analysis in section 4.4 describes a pure pooling equilibrium where the announcement $\hat{a}$ does not reveal any information to the investors. Section 4.5 describes partial pooling equilibria where the announcements partially reveal information to the investors. My model has no uncertainty after time 1. In equilibrium, after observing θ, anticipating the investors' equilibrium response $V_1(\hat{a})$, and choosing $\hat{a}$ and a at time 1; the firm manager can predict the cash flow $Z(\theta, \hat{a}, a)$ and the payoffs with certainty.

3.2. Manager's private information

The two divisions, ESTABLISHED and DEVELOPMENT, produce cash flow that will be realized at time 2. The production function for each division is the resources allocated to the division multiplied by that division's productivity parameter. See section 3.3 and assumption (A-2) for a description of the firm's production function.

ESTABLISHED operates in a stable mature industry where investors can accurately predict productivity. For example, customer loyalty in a mature industry may be so well-established that financial analysts can obtain market research reports that accurately predict market share and profits for every firm. In the ESTABLISHED industry any information available to firm managers is also available to investors. Thus, the model assumes the productivity parameter of ESTABLISHED is common knowledge.

DEVELOPMENT operates in a rapidly changing industry where the investors cannot obtain independent estimates of productivity. In DEVELOPMENT's industry financial statements of past performance are not accurate predictors of future performance. Firm managers observe relevant information that is not available to investors. For example, the firm manager may receive confidential reports of customer demand, competitor strategy, or engineering studies that are not available to investors. Thus, the productivity parameter of DEVELOPMENT is privately observed by the firm manager at time 1, and is referred to as the firm's type.

The firm's type is a realization θ from the random variable, $\tilde{\theta}$. The prior probability distribution of $\tilde{\theta}$ is common knowledge, but the realization θ is privately observed by the manager at time 1. Following Crawford and Sobel [1982] and Newman and Sansing [1993], this model assumes the type is uniform on the interval [0, 1]. Assuming a uniform distribution on $[0,1]$ makes the analysis more tractable. This assumption is labeled (A-1).

$$\tilde{\theta} \sim U(0,1) \qquad \text{(A-1)}$$

The productivity parameter for ESTABLISHED is fixed at one-half, the mean value of $\tilde{\theta}$. Thus, prior to privately observing θ, the manager expects both divisions to have a productivity parameter equal to one-half. An observation of θ greater than one-half can be interpreted as "private good news," because the firm's type is better than expected. If the firm manager of a high θ firm allocates more resources to DEVELOPMENT, then the firm will produce more cash than an average firm. Conversely, an observation of θ less than one-half can be interpreted as "private bad news," because the firm's type is worse than expected.

This model is contrived so that firm managers have an incentive to induce investors to believe the firm has a high type. Section 3.4 discusses the assumption that the manager's compensation depends on $V_1(\hat{a})$, the investors' valuation of the firm at time 1. Section 3.5 discusses the assumption that investors use the announcement $\hat{a}$ to revise their beliefs about firm type and determine the valuation $V_1(\hat{a})$. If the firm has a high type, represented by a value of θ greater than one-half, then the productivity of DEVELOPMENT is higher than ESTABLISHED. A result in chapter 4 shows that the highest type firm (θ=1) moves resources from ESTABLISHED to DEVELOPMENT, represented by an a greater than one-half, and announces the move of resources to DEVELOPMENT, represented by an $\hat{a}$ greater than one-half.

3.3. Firm's valuation function at time 2

This section specifies two functions. $Z(\theta, \hat{a}, a)$ is the cash available for distribution at time 2 and is specified as (A-2). $V_2(\theta, \hat{a}, a, V_1)$ is the residual value of the firm to the investors at time 2 and is specified as (A-3).

$Z(\theta, \hat{a}, a)$ is essentially the firm's production function. The firm's cash available for distribution at time 2 is defined by the function $Z(\theta, \hat{a}, a)$, and consists of three components. The first component is the cash produced by the resources allocated to the two operating divisions. The second component is the restructuring costs incurred by moving resources from one division to the other. The third component is a penalty for lying that is incurred if the firm manager makes an announcement $\hat{a}$ that differs from the actual resource allocation a. Each of these three components is discussed below and the complete function $Z(\theta, \hat{a}, a)$ is specified by assumption (A-2) at the end of the discussion.

The production function of each operating division is the product of its productivity parameter multiplied by the resources allocated to the division. DEVELOPMENT has productivity θ, receives resources a, and produces cash flow of θa. ESTABLISHED has productivity parameter equal to one-half, receives resources 1-a, and produces cash flow of $\frac{1}{2}(1-a)$. The total operating cash flow to the firm is the sum of both divisions' cash flow, $\theta a + \frac{1}{2}(1-a)$. Given the model's restrictions that a is restricted to the interval $[0,1]$ and θ is also restricted to $[0,1]$, the feasible total production for the two divisions ranges from zero to one.

The firm incurs restructuring costs in moving resources from one division to another. For example, the cost to shut down or reduce a division may include employee severance benefits, plant and equipment disposal costs, and settlement of claims with government regulators. Moving resources to a division may include employee relocation benefits, commissions to employee recruiters, training, new construction, remodeling, and licensing fees. Restructuring costs increase as the firm moves more resources away from the initial allocation of one-half to each division.

In this model I assume restructuring costs are determined by the quadratic function $k\left(a-\frac{1}{2}\right)^2$. The constant k is an exogenous positive parameter.

Section 4.2 analyzes the model and explains why the results become more tractable by fixing parameter k equal to one-half. The restriction k=0.5 is labeled assumption (A-7) in section 4.2. I analyzed the model without the restriction k=0.5 and found no fundamental change in results by restricting this parameter. Given the feasibility constraint that restricts a to the interval $[0,1]$, and the restriction k=0.5, the restructuring cost, $k\left(a-\frac{1}{2}\right)^2$, varies from zero to $\frac{1}{8}$. If the manager decides to leave all resources in their initial allocation, then a is one-half, and restructuring costs are zero.

A quadratic function for restructuring costs is assumed so that the choice of a that maximizes $Z(\bullet)$ is strictly in the interior of the $a \in [0,1]$ interval for most types. If the production restructuring costs were linear functions of a, then the optimal choice of a for most types would be at the upper boundary (a=1), the lower boundary (a=0), or indifference over the [0, 1] interval.

After observing θ and picking the strategy pair $\{\hat{a}, a\}$, the firm manager has no uncertainty about the cash flow $Z(\theta, \hat{a}, a)$. Adding some uncertainty to the production function would not change the analytical results, because the manager is assumed to be a risk-neutral expected compensation maximizer.

The timeline in Figure 2 shows the firm simultaneously chooses an announcement $\hat{a}$ and an allocation a at time 1. Lying in this model occurs if the firm's announcement $\hat{a}$ differs from its allocation a. Although the firm picks the allocation a at time 1, the actual movement of resources occurs during the operating cycle between time 1 and time 2. Financial auditors can not independently verify the actual resource allocation until time 2. When the firm issues audited financial statements at time 2, the actual resource allocation a is publicly revealed, and investors can compare a and $\hat{a}$.

The amount of lying in this model is the difference between the announcement $\hat{a}$ at time 1 and the allocation a revealed at time 2. I model the penalty for lying with the quadratic function $\hat{k}(\hat{a} - a)^2$. The parameter $\hat{k}$ is an exogenous constant. The analysis in chapter 4 shows the equilibria depend on the value of $\hat{k}$. With this function, the penalty amount approaches zero as the amount of lying approaches zero. A quadratic function rather than a linear function is assumed so that there are tractable solutions where the firm manager's optimal choice of $\hat{a}$ is strictly in the interior of the interval [0, 1] for some types.

In my model I assume any lying is always revealed and subject to penalty with certainty. Alternatively, I could have assumed that lying is detected with probability p, penalized at rate ϕ if detected, and zero penalty if not detected. In this alternative formulation the expected penalty rate would be the multiplicative product, $p\phi$; whereas my model represents the penalty rate by a single constant $\hat{k}$. Adding uncertainty to the detection process might better represent the real world, but it would not change the character of the analytical results since I assume the firm manager and investors are risk-neutral. If there were uncertainty, the firm manager would maximize his *expected* compensation at time 1. The effect of decreasing the detection probability p is equivalent to reducing the penalty rate $\hat{k}$.

In my model the penalty for lying directly reduces the firm's cash flow $Z(\bullet)$ and that reduces the total resources that could be distributed to the manager or investors. I model the penalty as a deadweight loss rather than a transfer payment from the firm to the investors who suffered losses. This implies the penalty in my model represents fees paid to attorneys, witnesses, consultants, courts, and regulators. If the firm manager were contractually committed to a policy of no lying, then the lying penalty would be avoided and the actual cash flow, $Z(\bullet)$, and the investors' expectation, $V_1(\hat{a})$, would be maximized. The expected firm manager's compensation is reduced as the expected amount of lying increases. A result of the analysis in Chapter 4 is that for some low type firms the benefits from lying are greater than the penalty. Firms with above average type would improve their payoffs if they could precommit all firm managers to a policy of no lying. Proposition 3, a result in Chapter 4, shows that at time 1 the firm managers who know they are low types could improve their payoff by deviating from the proposed agreement to tell the truth. I do not assume the investors are allowed to impose a contract that motivates firm managers to tell the truth.

The complete functional form of $Z(\bullet)$ is specified by assumption (A-2). The first two terms of (A-2) are the operating cash flows from DEVELOPMENT and ESTABLISHED, respectively. The third and fourth terms are the restructuring costs and the penalty for lying, respectively.

$$Z(\theta, \hat{a}, a) \equiv \theta a + \frac{1}{2}(1-a) - k\left(a - \frac{1}{2}\right)^2 - \hat{k}(\hat{a} - a)^2 \qquad \text{(A-2)}$$

where parameters $k > 0$ and $\hat{k} > 0$ and $\hat{a} \in [0, 1]$ and $a \in [0, 1]$

The parameter restrictions in (A-2) are discussed elsewhere and are summarized as follows. The restructuring cost rate k is restricted to k=0.5 at assumption (A-7) in section 4.2. The positive penalty rate $\hat{k}$ is justified by Rule 10b-5. The analysis in chapter 4 shows the character of the equilibrium results are sensitive to changes in the value of $\hat{k}$. Restricting $\hat{a}$ and a to the interval $[0,1]$ is a feasibility restriction on the firm manager's strategy choice that implies a fixed amount of resources can be allocated among the operating divisions.

The total firm cash value available at time 2 for distribution is Z, which is specified by the production function $Z(\theta, \hat{a}, a)$. The manager is paid a wage W, which is specified by the compensation function $W(\theta, \hat{a}, a, V_1)$ described in section 3.4. The investors receive the residual value V_2, as specified by the function $V_2(\theta, \hat{a}, a, V_1)$ defined by assumption (A-2).

$$V_2(\theta, \hat{a}, a, V_1) \equiv Z(\theta, \hat{a}, a) - W(\theta, \hat{a}, a, V_1) \qquad \text{(A-3)}$$

This model assumes the functional form of $V_2(\bullet)$, $W(\bullet)$, and $Z(\bullet)$, and parameter values k and $\hat{k}$ are common knowledge. At time 1 investors can observe $\hat{a}$, but not the firm type θ nor the operating decision a. Investors have expectations for $V_2(\bullet)$, as a function of their beliefs about the firm's type and operating decision a. Sections 3.5 and 3.6 show how the investors revise their expectation of $V_2(\bullet)$ after observing $\hat{a}$.

3.4. Manager's compensation function

In this model I follow the assumption of some prior financial disclosure models [Dye 1988; Miller and Rock 1985; Teoh and Hwang 1991] that the shareholders have conflicting consumption preferences across time. Some shareholders will sell their stock at time 1 and want the manager to maximize the firm value at time 1. Other shareholders plan to hold their stock to the end of the operating cycle, and want the manager to maximize firm value at time 2. The compensation contract is a compromise with some positive weight on firm value at both time 1 and time 2.

Since the purpose of this model is to characterize the manager's announcements rather than to identify the optimal agency contract, I assume the compensation contract is determined exogenously. The sharing of risk between managers and investors is not an issue in this paper, and I assume the manager is a risk-neutral compensation-maximizer. I assume the compensation contract is a linear combination of firm value at time 1 and time 2 with positive weight at both time points. A similar linear compensation function is specified in Miller and Rock [1985] and Teoh and Hwang [1991]. The analysis in chapter 4 shows that this contract induces the manager to make a tradeoff between maximizing firm value at time 1 and time 2.

The model assumes the compensation function $W(\bullet)$ is a linear combination of $V_1(\hat{a})$ and $V_2(\bullet)$ as defined in (A-4). Lemma 1 discussed in section 3.5 derives an alternative formula for $W(\bullet)$ as a function of $V_1(\hat{a})$ and $Z(\bullet)$.

$$W(\theta, \hat{a}, a, V_1(\hat{a})) \equiv \beta_1 V_1(\hat{a}) + \beta_2 V_2(\theta, \hat{a}, a, V_1(\hat{a})) \quad \text{(A-4)}$$

where $1 > \beta_1 > 0$ and $1 > \beta_2 > 0$

This compensation function could be implemented by a package of stock grants for the manager. Since the manager's stock ownership and options are required to be disclosed in the firm's proxy statement, it is assumed the functional form of $W(\bullet)$ and parameter values β_1 and β_2 are common knowledge.

The relative magnitude of β_1 and β_2 represent the relative weight in the manager's decision problem at time 1. However, no compensation is paid until time 2.

I assume that the manager is contractually bound to the firm from time 1 to time 2. If a manager resigned before the outcomes were revealed at time 2, then β_2 is zero and that manager could avoid the adverse consequences of the penalty for lying.

If $\beta_2 = 0$ and $\beta_1 > 0$, the penalty at time 2 has no effect on manager's compensation, and the managers would lie in the direction that maximized the stock price $V_1(\hat{a})$. As β_2 increases, the penalty at time 2 is more costly to the manager. A result in section 4.5.5 shows that as β_1 increases, lying increases and less information is disclosed to investors. Miller and Rock [1985] has a similar result where increasing the compensation function's weight on current rather than long-term share price results in less informative disclosure to investors. Increasing the incentive on current share price benefits shareholders who sell currently, but may reduce firm value for long-term shareholders.

If β_1 equals zero, the firm manager's incentives are perfectly aligned with maximizing shareholder at time 2, and there is no incentive to manipulate price at time 1. To maximize the cash at time 2, the firm will avoid penalties for lying and make a truthful announcement, $\hat{a} = a$. In the special case where $\beta_1 = 0$ and $\beta_2 = 1$, the manager and investors each receive half of the firm's available cash. This result can be shown by the following algebra.

Special case when $\beta_1 = 0$ *and* $\beta_2 = 1$:

$$W(\bullet) = 0 + V_2(\bullet)$$

$$V_2(\bullet) = Z(\bullet) - W(\bullet) = Z(\bullet) - V_2(\bullet)$$

$$V_2(\bullet) = \frac{Z(\bullet)}{2} = W(\bullet)$$

The numerical examples in chapter 4 assume small positive values for β_1 and β_2, since incentive compensation paid to most executives is small relative to the total market value of their corporations' stock [Jensen and Murphy 1990]. When β_1 and β_2 are small, the total compensation is small relative to total firm value $Z(\bullet)$.

3.5. Investors' valuation function at time 1

Since the focus of this model is the firm manager's announcement and allocation strategy, the investors' role is simply to act as a nonstrategic valuation mechanism at time 1. The investors observe the firm's announcement $\hat{a}$ at time 1 and revise their expectations about $V_2(\bullet)$, the firm's residual value at time 2. The expectation of $V_2(\bullet)$ depends on the investors' revised beliefs about the firm type.

The investors' belief revision is a rational expectations process [Lucas 1978; Muth 1961]. A similar nonstrategic role for investors has been adopted by many prior financial disclosure models in the accounting and finance literature [e. g., Miller and Rock 1985; Newman and Sansing 1993]. My model does not include other strategic players, such as competitors, who use the firm's announcement to take actions that affect firm value.

To maintain the focus on the manager's strategy, I follow the prior financial disclosure model literature and make several simplifying assumptions about investor behavior. The investment market is perfectly competitive. I assume all investors are risk-neutral, observe the same information, and have the same belief revision process. The investors' purpose in the model is to estimate firm value after the announcement at time 1. Investors are not allowed to make binding commitments with other investors. Investors cannot modify the manager's compensation contract. In equilibrium the investors' average profits are zero.

In this model investors observe the restructuring announcement of a single firm. I do not model how investors revise their expectations when they can view the announcements of many firms. I assume each firm manager is motivated to maximize his own compensation, and there is no central authority that can control announcement of all firms. If there were such a central authority, the sequence of events would call for investors to react after observing a vector of announcements by many firms, and a model in the style of Green and Laffont [1990] would be appropriate.

Prior to observing the announcement, investors believe the firm's type is uniform on $[0,1]$, as specified at assumption (A-1). After the announcement the investors' beliefs are denoted $\mu(\tilde{\theta}|\hat{a})$. The related probability density function is denoted $d\mu(\tilde{\theta}|\hat{a})$. This notation, $\mu(\tilde{\theta}|\hat{a})$ and $d\mu(\tilde{\theta}|\hat{a})$, is similar to the notation in Reinganum and Wilde [1986, p. 743].

The function $\mu(\tilde{\theta}|\hat{a})$ maps a feasible announcement $\hat{a} \in [0,1]$ into a distribution of the random variable $\tilde{\theta}$. For example, $\mu(\tilde{\theta}|\hat{a}=0.5) = \tilde{\theta} \sim U(0,0.5)$ indicates that if investors observe $\hat{a} = 0.5$, they revise their belief about the distribution of type from uniform on $[0,1]$ to uniform on $[0,0.5]$. One requirement of the equilibrium described in section 3.6 below is that the investors' revised beliefs must be consistent with the firm's equilibrium strategy.

The related probability density function, $d\mu(\tilde{\theta}|\hat{a})$, is a mapping into the non-negative real numbers. For example, if $\mu(\tilde{\theta}|\hat{a}=0.5) = \tilde{\theta} \sim U(0,0.5)$, then $d\mu(\tilde{\theta}|\hat{a}=0.5) = 2$ for $\theta \in [0, 0.5]$. This does not imply that the true value of θ cannot be greater than 0.5. Rather, this statement means the investors assign zero probability to the event that $\theta > 0.5$. In this example the investors' expectation of type is 0.25, the mean of the interval $[0,0.5]$.

$$E[\tilde{\theta}|\hat{a}=0.5,\ \mu(\bullet)] \;=\; \int_0^1 \theta\, d\mu(\tilde{\theta}|\hat{a}=0.5) \;=\; \int_0^{0.5} \theta\, 2\, d\theta \;=\; 0.25$$

In addition to making an inference about the firm's type, the investors infer the firm's allocation a. In the equilibrium described in section 3.6 below, the firm manager's equilibrium allocation rule is $a^e(\theta)$. In equilibrium, the investors know the allocation function is $a^e(\theta)$, even though they do not directly observe the realized type θ. The investors' expectation of the allocation is

$$\mathrm{E}_{\tilde{\theta}}\left[a^{e}(\tilde{\theta})\middle|\hat{a},\mu(\tilde{\theta}|\hat{a})\right] = \int_{0}^{1} a^{e}(\tilde{\theta})\, d\mu(\tilde{\theta}|\hat{a}).$$

The analysis could be modified to allow investors to believe some firm type mixes among several allocations. However, it is unreasonable to assume a firm would randomize among allocations when the choice of allocation directly effects firm cash flow, $Z(\bullet)$, and the manager's compensation, $W(\bullet)$. Given the quadratic production and penalty functions, the firm has a unique allocation that maximizes its payoff.

The investors' payoff at the end of the game is the firm's residual value, $V_2(\bullet)$, defined by assumption (A-3) in section 3.3. The investors' expectation of $V_2(\bullet)$ at time 1 given their beliefs is taken with respect to the random variable $\tilde{\theta}$ and is denoted $\mathrm{E}_{\tilde{\theta}}\left[V_2(\bullet)\middle|\hat{a},\mu(\tilde{\theta}|\hat{a})\right]$.

The price the investors pay for the firm at time 1 is the expectation at time 1 of residual value at time 2 discounted by a present-value factor. The pricing function at time 1 is $V_1(\hat{a}) = \frac{1}{1+\rho}\mathrm{E}_{\tilde{\theta}}\left[V_2(\bullet)\middle|\hat{a},\mu(\tilde{\theta}|\hat{a})\right]$, where ρ is the investors' discount rate between time 1 and time 2.

Choosing a nonzero discount rate does not add significant new insights to the model. Suppose a positive discount rate, $\rho > 0$, is assumed. Substituting the investors' discounted expectation into the manager's compensation function defined at (A-4) gives $W(\bullet) = \beta_1 V_1(\hat{a}) + \beta_2 V_2(\bullet) = \frac{\beta_1}{1+\rho}\mathrm{E}_{\tilde{\theta}}\left[V_2(\bullet)\middle|\hat{a},\mu(\tilde{\theta}|\hat{a})\right] + \beta_2 V_2(\bullet)$.

The essential feature of the model is manager's tradeoff between maximizing investors' expectations at time 1, denoted $V_1(\hat{a}) = \mathrm{E}_{\tilde{\theta}}\left[V_2(\bullet)\middle|\hat{a}, \mu\left(\tilde{\theta}\middle|\hat{a}\right)\right]$, or maximizing the actual outcome $V_2(\bullet)$. The relative weights on these values are $\frac{\beta_1}{1+\rho}$ and β_2, respectively. An increase in the discount rate ρ has the same effect on $\frac{\beta_1}{1+\rho}$ as a proportionate decrease in β_1. I fix the discount rate at ρ equal to zero, and leave β_1 as an exogenous parameter. A change in β_1 can be interpreted as a change in the relative power of shareholders at time 1 or a change in the discount rate over time.

In summary, the investors' price at time 1 equals their expectations of firm's residual value at time 2. The only random variable in this model is $\tilde{\theta}$ which is uniform on the interval [0,1]. The investors' belief about the firm's type after observing the announcement is represented by the posterior probability density function $d\mu\left(\tilde{\theta}\middle|\hat{a}\right)$. The function $a(\theta)$ maps the investors' inference about type and the observed announcement to an inference about the firm's allocation. These relationships are defined symbolically by assumption (A-5).

$$V_1(\hat{a}) \equiv \mathrm{E}_{\tilde{\theta}}\left[V_2(\bullet)\middle|\hat{a}, \mu\left(\tilde{\theta}\middle|\hat{a}\right)\right] = \int_0^1 V_2\left(\tilde{\theta}, \hat{a}, a\left(\tilde{\theta}\right), V_1(\hat{a})\right)\, d\mu\left(\tilde{\theta}\middle|\hat{a}\right) \qquad \text{(A-5)}$$

Inspection of (A-5) shows that $V_1(\hat{a})$ appears on both the left and right-hand-side of the equation. Lemma 1 in the Appendix derives an equivalent expression such that $V_1(\hat{a})$ appears only on the left-hand side. Result (1) of Lemma

1 gives the investors' valuation response as a function of their expectations of the cash available for distribution. Result (2) of Lemma 1 gives the manager's compensation as a function of the investors' expectation of cash flow and the actual cash flow.

Lemma 1. Given assumptions (A-3), (A-4), and (A-5),

$$V_1(\hat{a}) = \frac{E_{\tilde{\theta}}\left[Z(\bullet)\middle|\hat{a}\right]}{1+\beta_1+\beta_2} \tag{1}$$

$$\text{and} \quad W(\bullet) = \frac{\beta_1}{1+\beta_2}\frac{E_{\tilde{\theta}}\left[Z(\bullet)\middle|\hat{a}\right]}{1+\beta_1+\beta_2} + \frac{\beta_2}{1+\beta_2}Z(\bullet) \tag{2}$$

Proof: See page 126 in the Appendix.

At time 0, prior to observing the firm's announcement, the investors' expectation of firm value is $V_o = E_{\tilde{\theta}}\left[V_1(\hat{a})\right] = \int_0^1 V_1(\hat{a})d\theta$. The market return at time 1 is the change in value relative to the initial price, and is denoted $R(\hat{a}) \equiv \frac{V_1(\hat{a}) - V_0}{V_0}$. By construction, the expected return is zero.

$$E_{\tilde{\theta}}\left[R(\hat{a})\right] = \frac{E_{\tilde{\theta}}\left[V_1(\hat{a})\right] - V_0}{V_0} = \frac{V_0 - V_0}{V_0} = 0$$

3.6. Equilibrium

My model depends on some standard assumptions for signalling games. The payoff functions, prior probability distribution of type, and sequence of the game are common knowledge. Payoffs are nontransferable and no binding commitments are allowed. A self-interested firm manager maximizes his payoff after observing his own firm type.

The solution concept used in this paper is a Bayesian Nash equilibrium as discussed in section 2.6. This solution concept ensures that the investors extract all available information about the firm's type from its announcement. In equilibrium, the players' conditional probabilistic beliefs about each other are self-confirming.

Notation for the equilibrium is defined as follows. The manager's equilibrium strategy is a vector $\{\hat{a}^e(\theta),\ a^e(\theta)\}$, with two components: an equilibrium announcement rule $\hat{a}^e(\theta)$ and an equilibrium allocation rule $a^e(\theta)$. The firm's announcement rule $\hat{a}^e(\theta)$ can also be expressed as a likelihood function, $g^e(\hat{a}|\theta)$, that denotes the probability a firm type θ announces $\hat{a}$ in equilibrium. Expressing the announcement rule as a likelihood function is helpful in formulating the investors' Bayesian revision process.

The set of feasible announcements is [0,1]. The set of in-equilibrium announcements is $\hat{A}^e$; and the set of out-of-equilibrium announcements is $\hat{A}^O$.

$$\hat{A}^e \equiv \{\hat{a}:\ \hat{a} \in [0,1] \text{ and } g^e(\hat{a}|\theta) > 0 \text{ for some } \theta \in [0,1]\}$$

$$\hat{A}^O \equiv \{\hat{a}:\ \hat{a} \in [0,1] \text{ and } g^e(\hat{a}|\theta) = 0 \text{ for all } \theta \in [0,1]\}$$

If all feasible announcements are observed in equilibrium, then $\hat{A}^O$ is empty.

The investors' equilibrium response $V_1(\hat{a})$ is a function that maps announcements into a stock price at time 1. After observing the firm's announcement at time 1, the investors' belief function $\mu(\tilde{\theta}|\hat{a})$ maps the announcement into a distribution on the random variable $\tilde{\theta}$. The conditional probability density function $d\mu(\tilde{\theta}|\hat{a})$ expresses the same idea as a mapping from the announcement into a probability density number on $[0,\infty)$. The function $a(\theta)$ maps the investors' inference about θ into an inference about the allocation. The investors' response is rational given their beliefs about firm type and allocation.

In this model the Bayesian Nash equilibrium requires the following four conditions to be satisfied.

C-1. Firm's compensation-maximizing strategy.

$$\{\hat{a}^e(\theta),\ a^e(\theta)\} = \underset{\hat{a},\ a}{\operatorname{argmax}}\, W(\theta,\ \hat{a},\ a,\ V_1(\hat{a}))$$

$$\text{subject to } \hat{a} \in [0,1] \text{ and } a \in [0,1]$$

$$g^e(\hat{a}|\theta) = \begin{cases} 1 & \text{if } \hat{a} = \hat{a}^e(\theta) \text{ is a unique solution to } \mathbf{C\text{-}1} \text{ for } \theta \\ [0,1] & \text{if } \hat{a} \text{ is one of several solutions to } \mathbf{C\text{-}1} \text{ that } \theta \text{ randomizes among} \\ 0 & \text{if } \hat{a} \text{ is not a solution to } \mathbf{C\text{-}1} \text{ for any } \theta \end{cases}$$

$$\hat{A}^e \equiv \{\hat{a}:\ \hat{a} \in [0,1] \text{ and } g^e(\hat{a}|\theta) > 0 \text{ for some } \theta \in [0,1]\}$$

$$\hat{A}^O \equiv \{\hat{a}:\ \hat{a} \in [0,1] \text{ and } g^e(\hat{a}|\theta) = 0 \text{ for all } \theta \in [0,1]\}$$

C-2. Investors' response to in-equilibrium announcements.

$$V_1(\hat{a}^e(\theta)) = \int_0^1 V_2\left(\tilde{\theta}, \hat{a}^e(\theta), a^e(\tilde{\theta}), V_1(\hat{a}^e(\tilde{\theta}))\right) d\mu(\tilde{\theta}|\hat{a}^e(\tilde{\theta})) \quad \text{for } \hat{a} \in \hat{A}^e$$

C-3. Investors' belief revision follows Bayes' Rule after in-equilibrium announcements.

$$d\mu\left(\tilde{\theta}\middle|\hat{a}\right)=\frac{g^{e}\left(\hat{a}\middle|\tilde{\theta}\right)d\mu\left(\tilde{\theta}\right)}{\int_{0}^{1} g^{e}\left(\hat{a}\middle|\tilde{\theta}\right)d\mu\left(\tilde{\theta}\right)} \text{ for } \hat{a}\in\hat{A}^{e}$$

$d\mu\left(\tilde{\theta}\right)=1$ is the investors' prior belief from assumption (A-1).

C-4. Investors' response to out-of-equilibrium announcements.

$$V_{1}\left(\hat{a}^{o}\right) = \int_{0}^{1} V_{2}\left(\theta, \hat{a}^{o}, a^{e}\left(\tilde{\theta}\right), V_{1}\left(\hat{a}^{o}\right)\right) d\mu\left(\tilde{\theta}\middle|\hat{a}^{o}\right) \text{ for } \hat{a}^{o}\in\hat{A}^{O}$$

where $\mu\left(\tilde{\theta}\middle|\hat{a}^{o}\right)$ and $a\left(\tilde{\theta}\right)$ are exogenously specified for $\hat{a}^{o}\in\hat{A}^{O}$.

The first two conditions, **C-1** and **C-2**, require that in equilibrium the firm and investors play mutual best-responses. **C-3** is the investors' Bayesian revision for in-equilibrium announcements. The first three conditions are similar to Crawford-Sobel and Newman-Sansing. **C-4** is the investors' response to out-of-equilibrium announcements. **C-4** is needed for my pure and partial pooling equilibria, because those equilibria do have out-of-equilibrium announcements.

C-1 implies the firm's announcement and allocation strategy maximizes the manager's compensation given the investors' response. The firm's strategy $\{\hat{a}^{e}(\theta),\ a^{e}(\theta)\}$ is a mapping from firm type into the set of feasible announcements and allocations. The likelihood function $g^{e}\left(\hat{a}|\theta\right)$ expresses the same idea as a mapping from pairs of announcements and types into the probability space [0,1].

The specification of $g^e(\hat{a}|\theta)$ allows the firm to pick a mixed strategy such that some firm types are indifferent among several announcements or allocations. Mixed strategies are typical of cheap talk models where there is no direct cost to announcements [e. g., Crawford and Sobel 1982, Newman and Sansing 1993]. However, in my model for almost all firm types the manager has strict preferences among announcements and allocations, because the choice of allocation and announcement directly effect the penalty amount, $\hat{k}(\hat{a} - a)^2$, which changes the cash flow $Z(\bullet)$ and the manager's compensation $W(\bullet)$.

In section 4.5 there are boundary types in the partial pooling equilibria that are indifferent among the strategies of adjoining intervals. If a type is exactly equal to an interval boundary, then that type is indifferent between a "truthful" and a "false" announcement. I assume those boundary types choose the truthful announcement rather than randomizing between the truthful and false announcements. Therefore, the analysis in chapter 4 excludes mixed strategy equilibria and considers only pure strategy equilibria. In a pure strategy equilibrium each firm type chooses only one announcement, $\hat{a} \in \hat{A}^e$ and

$$g^e(\hat{a}|\theta) = \begin{cases} 1 & \text{if } \hat{a} = \hat{a}^e(\theta) \\ 0 & \text{otherwise} \end{cases}.$$

The firm manager chooses the allocation rule $a^e(\theta)$ and announcement rule $\hat{a}^e(\theta)$ concurrently. The strategy pair $\{\hat{a}^e(\theta),\ a^e(\theta)\}$ is an equilibrium when the firm manager does not prefer to change either component. Given the allocation $a^e(\theta)$ and investors' response $V_1(\hat{a})$, the manager will not deviate from the equilibrium announcement $\hat{a}^e(\theta)$. Given $\hat{a}^e(\theta)$ and $V_1(\hat{a})$, the manager will not deviate from $a^e(\theta)$.

C-2 says investors price the firm at its expected value given the firm's in-equilibrium strategy and the investors' beliefs at **C-3**. When an in-equilibrium announcement is observed, the investors' belief revision process is given by **C-3**. In equilibrium the investors infer that the firm's allocation is given by function $a^e(\theta)$. The investors do not directly observe the firm's allocation, but they infer that the probability density of firm type is $d\mu(\tilde{\theta}|\hat{a})$, and then infer that the allocation follows the allocation rule $a^e(\theta)$. The investors' expectation of type is $\int_0^1 \tilde{\theta}\, d\mu(\tilde{\theta}|\hat{a})$. The expectation of allocation is $\int_0^1 a^e(\tilde{\theta})\, d\mu(\tilde{\theta}|\hat{a})$.

C-3 implies that investors correctly infer the interval containing the firm's type when an in-equilibrium announcement is observed. The prior probability density function $d\mu(\tilde{\theta})$ equals one, because (A-1) assumed $\tilde{\theta} \sim U(0,1)$. The investors' beliefs after observing the announcement $\hat{a}$ are a Bayesian revision of their prior beliefs using the observed announcement $\hat{a}$, and the likelihood that $\hat{a}$ will be observed when the manager uses his equilibrium announcement rule, $\hat{a}^e(\theta)$. Condition **C-3** is defined only for in-equilibrium announcements, $\hat{a} \in \hat{A}^e$.

C-4 gives the investors' response to out-of-equilibrium announcements, $\hat{a}^o \in \hat{A}^O$. To determine whether the firm will deviate to an out-of-equilibrium announcement, the investors' response must be defined for all feasible announcements $\hat{a} \in [0,1]$. The investors' beliefs about firm type given an out-of-equilibrium announcement cannot be determined endogenously. For equilibria with a nonempty set of out-of-equilibrium announcements, I exogenously specify the investors' out-of-equilibrium beliefs, $d\mu\left(\tilde{\theta}\middle|\hat{a}^o\right)$ for $\hat{a}^o \in \hat{A}^O$. This posterior probability density function is well-defined so that $\int_0^1 d\mu\left(\tilde{\theta}\middle|\hat{a}^o\right) = 1$ for all out-of-equilibrium announcements, $\hat{a}^o \in \hat{A}^O$. The investors' belief about the firm's allocation a is given by $a(\theta)$ as a function of the inferred type θ. **C-4** requires the investors' response to an out-of-equilibrium announcement to be an expectation with respect to their posterior beliefs, $d\mu\left(\tilde{\theta}\middle|\hat{a}^o\right)$ for $\hat{a} \in \hat{A}^O$. **C-2**, **C-3**, and **C-4**, taken together, require that the investors' response to any announcement is sequentially rational given their revised beliefs.

3.7. Investors' Out-of-Equilibrium Beliefs

The pure and partial pooling equilibria discussed in sections 4.4 and 4.5, respectively, are characterized by a countable set of discrete in-equilibrium announcements. Since the feasible space of announcements is the continuous interval [0, 1], the pure and partial pooling equilibria have an infinite set of out-of-equilibrium announcements. To prove the existence of these equilibria, I need to show that no type has an incentive to deviate to any out-of-equilibrium announcement.

The investors' response to an announcement is a function of their beliefs about firm type. The investors' beliefs after observing the announcement $\hat{a}$ are denoted by the posterior probability density function $d\mu\left(\tilde{\theta}\middle|\hat{a}\right)$. Recall that assumption (A-5) in section 3.5 specifies the investors' valuation at time 1 is a function of $d\mu(\tilde{\theta}|\hat{a})$, $V_1(\hat{a}) \equiv \mathrm{E}_{\tilde{\theta}}[V_2(\bullet)|\hat{a},\mu(\bullet)] \equiv \int_0^1 V_2(\bullet)d\mu(\tilde{\theta}|\hat{a})$. The investors' posterior beliefs must be defined for all feasible $\hat{a}$, whether they are in or out of equilibrium.

If I allowed an infinite variety of out-of-equilibrium beliefs, there would be far too many equilibria to analyze. Prior signalling models have limited the number of equilibria by restricting out-of-equilibrium beliefs. In a sequential market entry game with two types of monopolists, strong and weak, Kreps and Wilson [1982, p. 263] restrict out-of-equilibrium beliefs so that entrants believe the probability of a strong monopolist is (weakly) greater if the monopolist is observed fighting entry.

Green and Laffont [1990, p. 257] adopt a similar plausibility restriction in a game with continuous types by assuming the incumbent's action rule $x^*(\theta)$ is a strictly monotonic increasing function of its type θ. Green-Laffont consider an example where $x < x'$, and there is an equilibrium where x' is observed in equilibrium and x is not used in equilibrium. In this example, if the incumbent deviated to x, then the attacker (entrant) believes that the type deviating to x is lower than the type announcing x'.

I adopt a similar monotonic restriction on out-of-equilibrium beliefs for the analysis in the next chapter. I assume investors believe the firm's announcement rule, $\hat{a}(\theta)$, is a weakly monotonic increasing function of its type. Higher type firms in this model with $\theta > 0.5$ have better prospects in DEVELOPMENT than in ESTABLISHED, and can increase firm value at time 2 by moving resources to DEVELOPMENT. To maximize firm value at time 2, the higher type firms should move resources from ESTABLISHED to DEVELOPMENT, denoted by $a > 0.5$, and make a truthful announcement, $\hat{a} = a > 0.5$, to avoid the penalty for lying. In contrast, lower type firms with $\theta < 0.5$ have better prospects in ESTABLISHED than DEVELOPMENT and can maximize firm value at time 2 by shifting resources in the other direction, $a < 0.5$. The compensation function motivates managers to maximize firm value at both time 1 and time 2. To increase $V_1(\hat{a})$ lower type firms have an incentive to mimic the higher type firms. Higher type firms do not have an incentive to mimic lower type firms.

Formally, I restrict the investors' posterior beliefs so that their posterior expectation of firm type is a weakly increasing function of the announcement. This restriction is symbolically stated by assumption (A-6)

$$\int_0^1 \theta d\mu\left(\tilde{\theta}\middle|\hat{a}'\right) = \mathrm{E}\left[\tilde{\theta}\middle|\hat{a}'\right] \leq \mathrm{E}\left[\tilde{\theta}\middle|\hat{a}''\right] = \int_0^1 \theta d\mu\left(\tilde{\theta}\middle|\hat{a}''\right) \quad \text{for any } \hat{a}' < \hat{a}'' \text{ and}$$

$$\mathrm{E}\left[\tilde{\theta}\middle|\hat{a}'\right] < \mathrm{E}\left[\tilde{\theta}\middle|\hat{a}''\right] \text{ for at least one pair } \{\hat{a}', \hat{a}''\} \text{ such that } 0 < \hat{a}' < \hat{a}'' < 1 \qquad \text{(A-6)}$$

The first part of assumption (A-6) means that if we select any two feasible announcements, $\hat{a}'$ and $\hat{a}''$, then the investors' expectation of type after observing the lower announcement is less than or equal to the expectation after observing the higher announcement. The second part of (A-6) means that investors' post-announcement expectation of type is not everywhere flat. For example, Figure 12 in section 4.5.3 shows the expectation in a partial pooling equilibrium is flat within an interval and makes a discontinuous upward jump at the upper end of each interval.

Note that (A-6) is a restriction about the investors' beliefs about *type*. (A-6) does not necessarily require that the investors' *pric*e response is a monotonic increasing function of the announcement. For example, in the partial pooling equilibrium of section 4.5, the expectation of type, $\mathrm{E}\left[\tilde{\theta}\middle|\hat{a}\right]$, is a weakly increasing function of $\hat{a}$, but the price response, $V_1(\hat{a})$, is not. Figure 13 in section 4.5 shows a discontinuous price response $V_1(\hat{a})$. In Figure 13, the price response is strictly increasing for in-equilibrium announcements, $\hat{a} \in \hat{A}^e$. However, the price response is not monotonically increasing for out-of-equilibrium announcements, $\hat{a} \in \hat{A}^o$.

CHAPTER 4

ANALYSIS

4.1. Introduction

This chapter analyzes the model and discusses equilibrium outcomes. Each section of this chapter considers different parameter settings of the basic model described in chapter 3.

Section 4.2 analyzes the model for the first-best information environment where the firm's type and allocation are publicly observable. The results of this section are a benchmark for the subsequent sections. Proposition 1 identifies the allocation rule, $a^*(\theta)$, that maximizes the firm's available cash at time 2, $Z(\bullet)$. Distortion is defined as an allocation that differs from the cash-maximizing allocation $a^*(\theta)$. Lying is defined as an announcement that differs from its allocation, $\hat{a} \neq a$. Proposition 2 shows that if the information environment is first-best, then the equilibrium outcome has all firms choosing no distortion and no lying.

Section 4.3 considers the second-best information environment where the firm privately observes it type θ, and privately chooses its allocation a. In this second-best environment, Proposition 3 shows there are always some firms who lie and mimic the announcements of higher types. When some firms lie, the investors cannot invert the announcement to fully reveal the firm's type. Corollary 3.2 shows as the penalty rate $\hat{k}$ approaches infinity, the outcome is not first-best.

Section 4.4 considers the second-best environment where the value of $\hat{k}$ is relatively *small*. Given these conditions, Proposition 4 shows there exists a pure pooling equilibrium such that all firms mimic the announcement of the highest type. In this pure pooling, all firms, except the highest type, are lying. Since all firms make the same announcement in a pure pooling equilibrium, that announcement reveals nothing to the investors. The investors' equilibrium response to the pure pooling is the expected price given that nearly all firms are lying.

Section 4.5 considers the second-best environment where the value of $\hat{k}$ is relatively *large*. Given these conditions, Proposition 5 shows there exists a partial pooling equilibrium in which the range of types is partitioned into a large number of intervals. Each interval is characterized by a unique announcement. When the investors observe an announcement in a partial pooling equilibrium, they can infer the interval of types that sent the announcement. The partial pooling gives the investors a partial revelation of the firm's type. Subsection 4.5.4 analyzes the partial pooling equilibrium in the special case where $\hat{k}$ is infinite. Subsection 4.5.5 considers the sensitivity of the interval boundaries in the partial pooling equilibrium to exogenous changes in the penalty rate, $\hat{k}$, and the compensation weighting factors, β_1 and β_2.

4.2. First-best case

This section considers the first-best case for this model which occurs if the firm's type and allocation are publicly observable at time 1. Subsequent sections of this chapter analyze the second-best environment where the firm's type and allocation are privately observed by the manager. The functions and definitions introduced in this section will be useful in the later sections.

As a benchmark, I begin by finding the strategy that would maximize the firm's available cash at time 2 without considering the manager's incentives or the investors' response. Since assumption (A-3) specified $Z(\bullet)$ as a quadratic function of firm type θ; the cash-maximizing allocation rule, $a^*(\theta)$, is a linear function of θ. Any announcement other than $\hat{a}^*(\theta) = a^*(\theta)$, would cause a positive penalty, $\hat{k}(\hat{a} - a)^2$, and decrease $Z(\bullet)$. The announcement rule $\hat{a}^*(\theta)$ is an increasing function of type, consistent with assumption (A-6) discussed at the end of chapter 3.

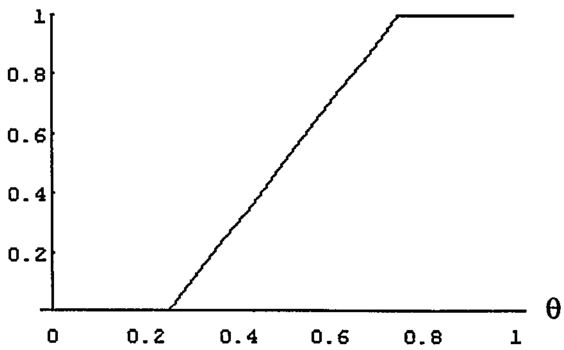
The above intuition is formalized in Proposition 1.

Proposition 1. Firm cash at time 2, denoted $Z(\theta, \hat{a}, a)$, is maximized when the firm chooses allocation $a^*(\theta) = \frac{1}{2} - \frac{1}{4k} + \frac{\theta}{2k}$ and announcement $\hat{a}^*(\theta) = a^*(\theta)$.

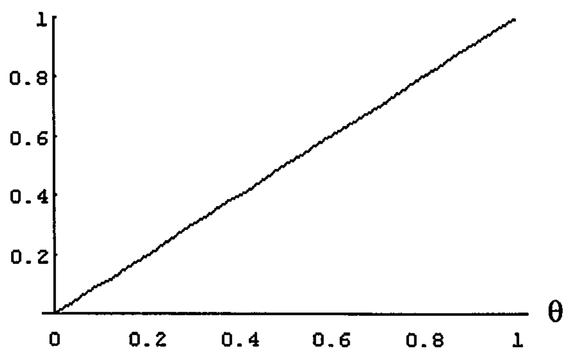
Proof: See Appendix at page 127.

Figure 3. Sensitivity of announcement strategy $\hat{a}^*(\theta)$ to parameter k.

$k=\frac{1}{4} \Rightarrow a^*(\theta)=\hat{a}^*(\theta)=-\frac{1}{2}+2\theta$ subject to $a^*(\theta)\in[0,1]$



$k=\frac{1}{2} \Rightarrow a^*(\theta)=\hat{a}^*(\theta)=\theta$



$k=1 \Rightarrow a^*(\theta)=\hat{a}^*(\theta)=\frac{1}{4}+\frac{1}{2}\theta$

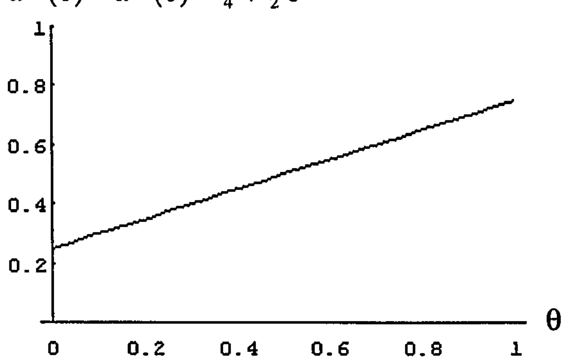


Figure 3 on the previous page illustrates how the cash-maximizing announcement strategy, $\hat{a}^*(\theta)$, is sensitive to the exogenous parameter *k*. When *k* is less than 0.5, the feasibility constraints $\hat{a} \le 1$ and $\hat{a} \ge 0$ are binding. For example, when *k*=0.25, there is an interval of types $\theta \in [0.75, 1]$ such that all types in the interval announce all resources will move to DEVELOPMENT, $\hat{a} = 1$. When *k*=0.5, the function $\hat{a}^*(\theta) = \theta$ is a one-to-one mapping from the feasible types $\theta \in [0,1]$ into the feasible announcements $\hat{a} \in [0,1]$. Thus, when *k*=0.5 each type chooses a unique announcement, and for every feasible announcement there is a a unique type. When *k* is greater than 0.5, each type has a unique announcement, but not all feasible announcements are observed. For example, when *k*=1, no types announce $\hat{a} > 0.75$.

This analysis in this paper becomes tractable when there exists an announcement rule that can be inverted to fully reveal the type. An announcement rule, $\hat{a}(\theta)$, is fully-revealing if the inverse $\Phi(\hat{a}(\theta))$ is a one-to-one mapping from the set of equilibrium announcements, $\hat{A}^e$, into a subset of feasible types, $\theta \in [0,1]$. Figure 3 shows the $\hat{a}^*(\theta)$ is not fully-revealing for values of *k* less than 0.5. Considering the cases of *k*<0.5 and *k*>0.5 significantly adds to the complexity of the proofs in the Appendix. Therefore, I limit the remainder of the analysis to cases where $\hat{a}^*(\theta)$ is fully-revealing and restrict *k* to one-half. This restriction is labeled $(A-7)$.

$$k = 0.5 \qquad \text{(A-7)}$$

Given assumption (A-7), the allocation rule $a^*(\theta)$ is a strictly increasing function of θ. If all firms adopt the allocation rule $a^*(\theta)$, then the highest type firm, θ=1, moves more resources from ESTABLISHED to DEVELOPMENT than any lower type firm.

Since the firm's cash production function $Z(\bullet)$ is common knowledge, both the investors and the firm manager can calculate $a^*(\theta)$. In the first-best case the investors know at time 1 if a firm's allocation deviated from $a^*(\theta)$. In the second-best case, the investors can not detect deviations until time 2.

Distinguishing between distortion and lying is important in this paper's analysis. The equilibrium analysis of the second-best case in the subsequent sections of this chapter shows the manager makes a tradeoff between distortion and lying in choosing his best-response.

Lying is defined as the difference between the announcement $\hat{a}$ and the allocation a. If a firms lies at time 1, then at time 2 the penalty for lying is $\hat{k}(\hat{a} - a)^2$.

Definition. A firm **lies** if its announcement differs from its allocation, $\hat{a} \neq a$. An outcome of **no lying** occurs if and only if $\hat{a}(\theta) = a(\theta)$ for all $\theta \in [0, 1]$.

Distortion occurs when there is at least one type that deviates from the allocation $a^*(\theta)$. If no distortion occurs, the cash at time 2 is $Z(\theta, \hat{a}, a^*(\theta))$. Suppose a firm chooses a distorted allocation, a^d, rather than its first-best allocation, a^*. Then the difference in allocation is $(a^* - a^d)$, and the change in available cash at time 2 is a nonlinear function of this difference. After substituting and simplifying,

$$Z(\theta, \hat{a}, a^*) - Z(\theta, \hat{a}, a^d) = (a^* - a^d)\left\{\theta - \frac{1}{2} - k(a^* + a^d - 1) - \hat{k}(a^* + a^d - 2\hat{a})\right\}$$

Definition. A firm of type θ **distorts** its allocation if it chooses $a \neq a^*(\theta)$. An outcome of **no distortion** occurs if and only if $a(\theta) = a^*(\theta)$ for all $\theta \in [0, 1]$.

Proposition 2 finds the equilibrium strategies for the firm and the investors in the first-best information environment. Proposition 1 identifies the $a^*(\theta)$ that maximizes $Z(\bullet)$, but does not prove that is the firm's equilibrium strategy in the first-best case. In the first-best case, the investors directly observe θ, a, and $\hat{a}$ at time 1 and can predict the cash at time 2, denoted $Z(\theta, \hat{a}, a)$, with certainty. From Proposition 1 investors and the firm know that to maximize the cash at time 2, the firm should choose the allocation $a^*(\theta)$. When θ, a, and $\hat{a}$ are publicly observable at time 1, the investors can directly observe any distortion or lying, and adjust their response $V_1(\hat{a})$.

Proposition 2. If the firm's type and allocation are publicly observable, the firm's equilibrium strategy is $\hat{a}(\theta) = a(\theta) = a^*(\theta) = \theta$ for all $\theta \in [0, 1]$, which is defined as no lying and no distortion.

Proof: See Appendix at page 129.

The proof of Proposition 2 assumes the penalty for lying in the first-best case is the same $\hat{k}(\hat{a} - a)^2$ as exists in the second-best case. Given this assumption the firm manager in the first-best case strictly prefers to tell the truth and avoid any penalty, denoted $a = \hat{a}$.

The outcome of the first-best case described by Proposition 2 is illustrated by Figure 4 on the next page. In this example, announcements and allocations are linear functions of type, $\hat{a}^*(\theta) = a^*(\theta) = \theta$. The choice of the highest type, θ=1, is $\hat{a}^*(1) = a^*(1) = 1$ which implies the highest type moves all resources from ESTABLISHED to DEVELOPMENT. The choice of the lowest type, θ=0, is $\hat{a}^*(0) = a^*(0) = 0$ which implies the lowest type moves all resources from DEVELOPMENT to ESTABLISHED. In this example the investor observes $\theta = a = \hat{a}$. The bottom of Figure 4 plots the investors' response $V_1(\bullet)$ as a function of the single variable $\hat{a}$. Because the production function $Z(\bullet)$ is quadratic, the investors' response $V_1(\hat{a})$ is a quadratic function of $\hat{a}$.

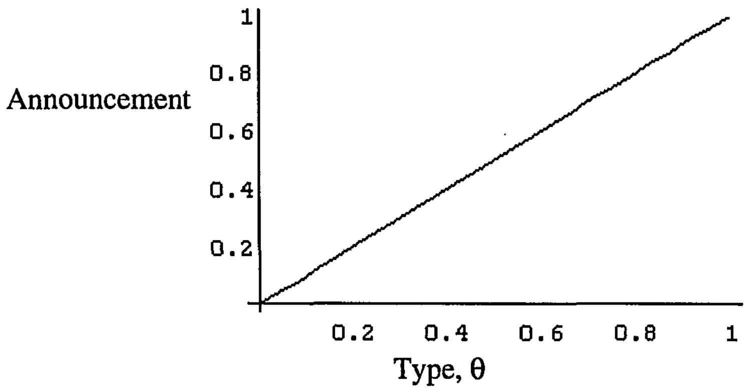
Figure 4. Outcome in first-best case.

Parameter values: $\beta_1 = 0.01,\ \ \beta_2 = 0.04,\ \ k = 0.5$

Results: $\hat{a}*(\theta) = a*(\theta) = \theta$

$$V_1*(\hat{a}) = V_1(\theta, \hat{a}*(\theta), a*(\theta)) = V_1(\theta,\theta,\theta) = 0.357 + 0.476\hat{a}^2$$

Plot of $\hat{a}*(\theta) = a*(\theta) = \theta$



Plot of $V_1*(\hat{a}) = 0.357 + 0.476\hat{a}^2$

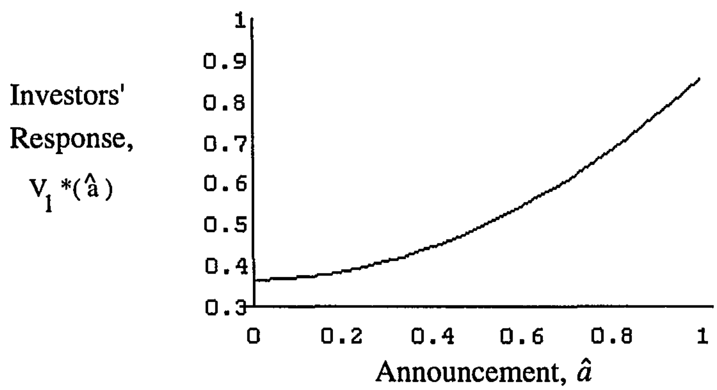


Figure 5 illustrates the investors' price change in the first-best case as a stock market return. V_0 * is defined as the stock market price of the firm at time 0, before the firm announces $\hat{a}$, with the expectation that the firm's strategy at time 1 will be $\hat{a}^*(\theta)$. At time 1, the investor revises the price to $V_1^*(\hat{a})$. The price return, $R^*(\hat{a})$, is calculated by dividing the difference in price by the initial price.

Figure 5. Market return in first-best case.

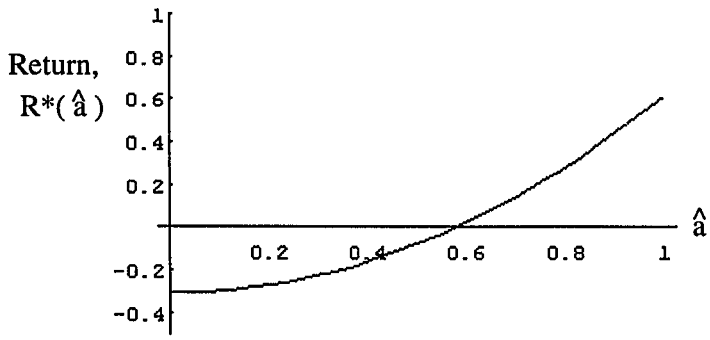
Parameter values: $\beta_1 = 0.01$, $\beta_2 = 0.04$, $k = 0.5$

Before observing $\hat{a}$, the expected price is

$$V_0^* = E_{\tilde{\theta}}\left[V_1^*\left(\hat{a}^*\left(\tilde{\theta}\right)\right)\right] = 0.357 + 0.476\frac{\hat{a}^3}{3}\Bigg|_{\hat{a}=0}^{\hat{a}=1} = 0.516.$$

After observing $\hat{a}$, price is $V_1^*(\hat{a}) = 0.357 + 0.476\hat{a}^2$.

Return is $R^*(\hat{a}) \equiv \dfrac{V_1^*(\hat{a}) - V_0^*}{V_0^*}$



The market return in the first-best case, given the parameter values in Figure 5, is an increasing function of the announcement. The price return is a positive 61.5 percent for the highest announcement, $\hat{a} = 1$; and a negative 30.8 percent for the lowest announcement, $\hat{a} = 0$. Figure 5 serves as a benchmark for examples in subsequent sections where the information environment is not first-best.

If the information environment is not first-best, then the investors observe the announcement $\hat{a}$ and make inferences about the firm's private information, type θ, and the firm's private action, allocation a. In a full revelation equilibrium, the investors can invert the announcement rule to fully reveal the type.

Definition. A **full revelation equilibrium** exists if and only if there exists an inverse function $\Phi(\hat{a})$ such that $\Phi(\hat{a}(\theta)) = \theta$ for all $\theta \in [0,1]$.

A full revelation announcement strategy may involve some lying. For example, suppose the firm's announcement strategy is $\hat{a}(\theta) = \sqrt{\theta}$ and its allocation strategy is $a(\theta) = \theta$. If this announcement-allocation strategy pair is used, then nearly every firm type lies, because $\hat{a}(\theta) - a(\theta) = \sqrt{\theta} - \theta > 0$ for $0 < \theta < 1$, and the penalty for lying is $\hat{k}(\sqrt{\theta} - \theta)^2$. When investors observe the announcement, they apply the inverse function $\Phi(\hat{a}) = \hat{a}^2$ to fully reveal the type θ. If this were an equilibrium, the investors' belief about the allocation would be consistent with the firm's allocation strategy, $a(\theta) = \theta$. The investors can use the inferred type, $\hat{a}^2$, to correctly infer the allocation equals $\hat{a}^2$; the penalty, $\hat{k}(\hat{a} - \hat{a}^2)^2$; and the future cash flow, $Z(\hat{a}^2, \hat{a}, \hat{a}^2)$. These inferences would be correct, and both the investors and firm would know future value with certainty. However, in this model full revelation with lying is not an *equilibrium* strategy for the firm.

The next analytical result of this section is that if a full revelation equilibrium exists in this model, then it must involve no lying, defined as $\hat{a}(\theta) = a(\theta)$ for all $\theta \in [0, 1]$. This result is driven by the model's assumption that lying is costly. Lying is beneficial to the firm if the investors raise their expectations of firm type and stock price $V_1(\hat{a})$ after observing a false announcement. Low types may raise expectations of investors by mimicking the announcements of a higher type. However, if the announcement strategy has both fully revelation and lying, as in the example in the previous paragraph, then the firm has a costly lying penalty with no offsetting benefit.

Corollary 2.1 states that if a full revelation equilibrium exists, then the firm's strategy is no lying and no distortion. The proof of Corollary 2.1 in the Appendix shows that if the announcement fully reveals type, the firm's compensation is maximized by a strategy of no lying and no distortion.

Corollary 2.1. A full-revelation equilibrium exists if and only if the firm's equilibrium strategy is $\hat{a}(\theta) = a(\theta) = a^*(\theta)$ for all $\theta \in [0, 1]$.

Proof: See Appendix at page 130.

Corollary 2.1 is useful in proving when a full revelation equilibrium does not exist. This corollary implies if a full revelation equilibrium exists, then the outcome is equivalent to the first-best outcome of no lying and no distortion. Corollary 2.1 is used in the next section to prove that a full revelation equilibrium does not exist when the firm privately observes its type.

4.3. Nonexistence of Full Revelation Equilibrium When Type Is Private

Sections 4.3, 4.4, and 4.5 analyze the incomplete information environment where at time 1 the firm privately observes its type θ and privately chooses its allocation a. Since the investors in this environment observe $\hat{a}$ but not a at time 1, they cannot detect the amount of lying at time 1. Undetected lying at time 1 may benefit firms by increasing its stock value at time 1, but will reduce stock value at time 2 when all lying is revealed. There are some firms for whom the benefits of lying at time 1 exceed the loss from the penalty at time 2.

Proposition 3 states that a full revelation equilibrium can not be sustained when firms privately observe type. The proof of Proposition 3 shows there exists some continuous interval M of types that will lie and mimic the announcement of the highest type, $\hat{a}^*(1)$. For all firms in interval M the benefit of increasing the stock value at time 1 is greater than the loss from the penalty at time 2. When the investors observe the announcement $\hat{a}^*(1)$, they know that type is contained in the interval M, but can not infer the exact value of θ. Proposition 3 is illustrated by example in Figure 6 on the next page.

Proposition 3. If the firm privately observes its type, then a fully revealing equilibrium does not exist.

Proof: See Appendix at page 132.

Figure 6. Deviation from full revelation equilibrium

Parameter values: $\beta_1 = 0.01$, $\beta_2 = 0.04$, $k = 0.5$, $\hat{k} = 0.25$

Results: $\hat{a}^*(\theta) = a^*(\theta) = \theta$

$$V_1^*(\hat{a}) = V_1(\theta, \hat{a}^*(\theta), a^*(\theta)) = V_1(\theta, \theta, \theta) = 0.357 + 0.476\,\hat{a}^2$$

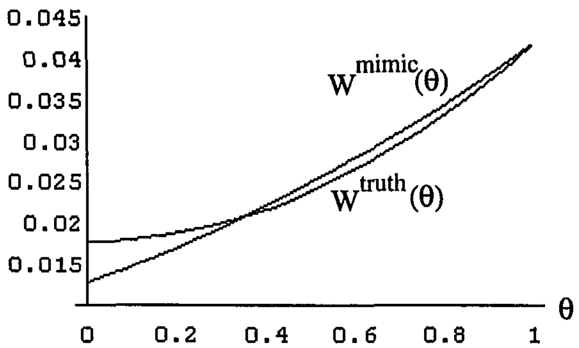
$$W^{\text{truth}}(\theta) = W(\theta, \theta, \theta, V_1^*(\theta)) = 0.005952\,(3 + 4\,\theta^2)$$

$$W^{\text{mimic}}(\theta) = W(\theta, 1, \theta, V_1^*(1)) = 0.003205\,(4 + 6\theta + 3\theta^2)$$

Solutions to $W^{\text{truth}}(\theta) = W^{\text{mimic}}(\theta)$ are $\theta = 0.355$ and $\theta = 1$

Interval $M = \{\theta : 0.355 < \theta < 1\}$

Plot of $W^{\text{truth}}(\theta)$ and $W^{\text{mimic}}(\theta)$



Benefit at time 1 of mimicking is $B(\theta) = 0.004579\,(1 - \theta^2)$

Loss at time 2 from mimicking is $L(\theta) = 0.03846\,\hat{k}(1-\theta)^2 = 0.009615\,(1-\theta)^2$

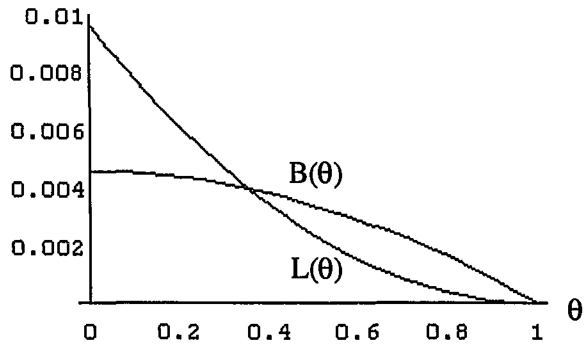


Figure 6 assumes investors respond as if they believe the firm's strategy is truthful. The investors' response as a function of these beliefs is $V_1^*(\hat{a})$. The announcement of the highest type, $\theta = 1$, is $\hat{a}^*(1) = 1$. The first plot compares the firm's payoff from two possible strategies. $W^{truth}(\theta)$ is the payoff to firm θ from the first-best strategy $\hat{a}^*(\theta) = a^*(\theta) = \theta$. $W^{mimic}(\theta)$ is the payoff to firm θ from the allocation $a^*(\theta)$ and mimicking the highest type's announcement $\hat{a} = 1$. The highest type announces $\hat{a}^*(1) = 1$ under both strategies and $W^{truth}(1) = W^{mimic}(1) = 0.04166$. Type θ=0.355 is indifferent between the two strategies and $W^{truth}(0.355)$ = $W^{mimic}(0.355) = 0.0209$. All types strictly between 0.355 and 1 strictly prefer to mimic. Since all of these types make the same announcement, $\hat{a} = 1$, the investors cannot distinguish whether the firm is actually $\theta = 1$ or some inferior type mimicking the highest type. Thus, full revelation is not an equilibrium outcome in this example.

The two plots in Figure 6 express the same idea with different functions. The first plot is in terms of the firm's total payoff from either strategy. The second plot in Figure 6 compares the benefits of mimicking, $B(\theta)$, to the losses from mimicking, $L(\theta)$. The benefits of mimicking are the increased stock value at time 1 weighted by the parameter β_1. The loss of mimicking is a function of the penalty for lying at time 2 weighted by the parameter β_2.

$$B(\theta) = \frac{\beta_1}{1+\beta_2}\left\{V_1^*(1) - V_1^*(\theta)\right\} = 0.004579\left(1-\theta^2\right)$$

$$L(\theta) = \frac{\beta_2}{1+\beta_2}\hat{k}(1-\theta)^2 = 0.009615\,(1-\theta)^2$$

The second plot shows the benefits and losses of mimicking are equal when $\theta = 0.355$, because $B(0.355) = L(0.355) = 0.00400$.

In many analytical models with discrete types, lying can be eliminated by increasing the penalty factors to a sufficiently high level. However, Corollary 3.1 shows that in this model with continuous types there is always some interval that strictly prefers to lie for any finite penalty factor.

Corollary 3.1. If the firm privately observes its type and $\hat{k}$ is exogenously increased to an arbitrarily large positive finite value, there always exists some interval of types that lie.

Proof: See Appendix at page 134.

The result in Corollary 3.1 can be understood by reviewing the previous numerical examples. Figure 4 shows that in a full revelation equilibrium the investors' response, $V_1{*}(\hat{a})$, is steepest as $\hat{a}$ approaches the feasible endpoint, $\hat{a} = 1$. This suggests that if a firm chooses to lie at time 1, then its greatest benefit at time 1 is to mimic the highest type. When the investors believe no firm lies, then the investors' response, $V_1{*}(\hat{a})$, does not depend on the penalty factor, $\hat{k}$. Changing from $\hat{a} < 1$ to $\hat{a} = 1$ results in an increase in investors' response $V_1{*}(\bullet)$ that is a quadratic function of $\hat{a} < 1$ and $\hat{a} = 1$. The penalty for announcing $\hat{a} = 1$ when allocation is $a = \theta$ is also a quadratic function of $\hat{a} < 1$ and $\hat{a} = 1$. The penalty is directly proportional to $\hat{k}(1-\theta)^2$, which approaches zero as type approaches $\theta = 1$. Recall that in this model the feasible type set is continuous. Thus, for any finite $\hat{k}$, there exists some type close to $\theta = 1$ for whom the penalty $\hat{k}(1-\theta)^2$ is arbitrarily close to zero. Figure 6 shows there exists an interval M with some types sufficiently close to $\theta = 1$ such that the benefits of mimicking, $B(\theta)$, exceed the loss from the penalty, $L(\theta)$. The supremum of interval M is fixed since $B(1) = L(1) = 0$. The infimum of M is the lower intersection where $B(\theta) = L(\theta)$. As $\hat{k}$ increases, the $L(\theta)$ function becomes steeper, $B(\theta)$ remains fixed, and the infimum of interval M shifts to the right.

The plot at the bottom of Figure 6 shows that both $B(\theta)$ and $L(\theta)$ decrease as θ increases towards $\theta = 1$. However, the curvatures of the two graphs differ. $B(\theta)$ is concave with respect to the origin. $L(\theta)$ is convex with respect to the origin.

Corollary 3.1 depends on the assumption that types are continuous (assumption (A-1)) and the penalty is a function of the magnitude of lying (assumption (A-3)). If the feasible types were drawn from a discrete set, the benefits for the second highest-type to mimic the highest type would be some fixed amount, x. By increasing the penalty rate, the loss from lying could be increased to some value greater than x. This penalty function, $\hat{k}(a - \hat{a})^2$, approaches zero as the magnitude of lying decreases to zero.

Lying is eliminated in this model when the penalty rate on lying, $\hat{k}$, becomes infinite. When lying is eliminated, all firm types choose an announcement equal to their allocation, $a(\theta) = \hat{a}(\theta)$ for all $\theta \in [0,1]$. However, eliminating lying does not eliminate distortion, the difference between the chosen allocation, $a(\theta)$, and the cash-maximizing allocation, $a^*(\theta)$.

For example, a firm with an average type, $\theta = 0.5$, has several feasible strategies with no lying. For this firm, the strategy with the first-best outcome is to announce no restructuring and leave all resources equally allocated between the two divisions. This first-best strategy for $\theta = 0.5$ is denoted $\{\hat{a} = 0.5, a = 0.5\}$. This first-best strategy is the cash-maximizing strategy for the firm with type $\theta = 0.5$. Another strategy with no lying is to mimic the highest type by announcing a transfer of all resources to DEVELOPMENT, and actually carrying out that restructuring. This mimicking strategy is denoted $\{\hat{a} = 1, a = 1\}$. Note that this mimicking strategy involves no lying since the announcement and allocation are equal. If the investors' valuation response at time 1 is higher after observing $\hat{a} = 1$ than after observing $\hat{a} = 0.5$, symbolized by $V_1(1) > V_1(0.5)$, then the mimicking strategy $\{\hat{a} = 1, a = 1\}$ gives the firm a higher value at time 1 and avoids any penalty for lying at time 2. For the firm with type $\theta = 0.5$ the mimicking strategy $\{\hat{a} = 1, a = 1\}$ distorts the allocation away the operating cash-maximizing first-best strategy $\{\hat{a} = 0.5, a = 0.5\}$. Depending on the values of the exogenous parameters, β_1 and β_2, the firm manager with type $\theta = 0.5$ may receive more compensation from the mimicking strategy than from its first-best strategy.

The intuition of the above example is generalized as Corollary 3.2. In Proposition 6 in section 4.5.4, I show that when $\hat{k}$ is infinite there exists a partial pooling equilibrium with no lying and a positive amount of distortion by almost all firms.

Corollary 3.2. If the firm privately observes its type and $\hat{k}$ approaches positive infinity, then the first-best outcome is not an equilibrium.

Proof: See Appendix at page 135.

4.4. Pure Pooling Equilibrium

When firms privately observe type, Proposition 3 showed some firms choose announcements that mimic higher types. The discussion of Corollary 3.1 showed there always exists an interval M so that types in M choose to mimic the highest type by announcing $a^*(1)$. If the interval M contains the entire set of feasible types [0,1], then a pure pooling occurs in which all types announce $a^*(1)$.

Proposition 4 states that there exist parameter values such that pure pooling is an equilibrium. The proof shows that no type deviates from announcing $a^*(1)$. This result is sensitive to the value of the penalty factor $\hat{k}$. If $\hat{k}$ is sufficiently small, then the benefits of mimicking exceed the costs for all types and a pure pooling equilibrium occurs. If $\hat{k}$ is sufficiently large, then the benefits of mimicking the highest type are less than the costs for some low types and pure pooling is not an equilibrium. If $\hat{k}$ is sufficiently large, then Proposition 5 in the next section shows there is a partial pooling equilibrium with many distinct pools.

Proposition 4. If the firm privately observes its type and the penalty factor $\hat{k}$ is sufficiently small, such that $0 < \hat{k} < \dfrac{\beta_1 k}{3\beta_2(1+\beta_1+\beta_2)}$, there exists a pure pooling equilibrium in which all types mimic the highest type.

Proof: See Appendix at page 139.

The pure pooling equilibrium of Proposition 4 is demonstrated by a numerical example and figures on the following pages. Figure 7 illustrates the firm's strategy in equilibrium. Figure 8 illustrates the investors' beliefs and response.

Figure 7. Firm's strategy in pure pooling equilibrium.

Parameter values: $\beta_1 = 0.01,\ \ \beta_2 = 0.04,\ \ k = 0.5,\ \ \hat{k} = 0.035$

Investors' posterior beliefs: $\mu(\tilde{\theta}|\hat{a}) = \tilde{\theta} \sim U(0,\hat{a})$

Results (rounded to 3 significant digits):

$$V_1^{\text{HP}}(\hat{a}) = 0.357 + 0.148\,\hat{a}^2$$

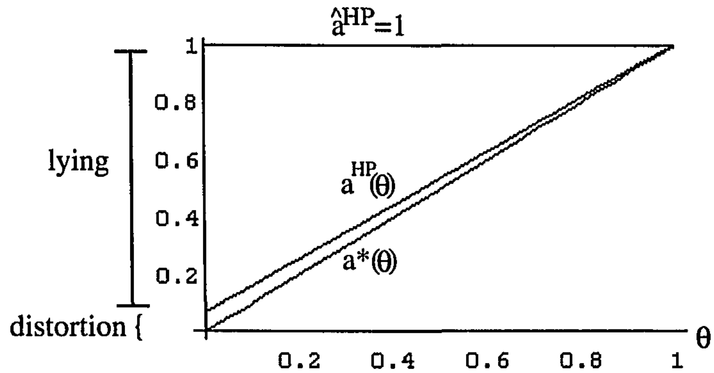
$$\hat{a}^{\text{HP}} = \hat{a}^*(1) = 1 \quad \text{for all } \theta \in [0,1]$$

$$a^{\text{HP}}(\theta) = \frac{k\,a^*(\theta) + \hat{k}a^*(1)}{k+\hat{k}} = 0.065 + 0.935\,\theta$$

$$a^*(\theta) = \theta$$

Lying: $\hat{a}^{\text{HP}}(\theta) - a^{\text{HP}}(\theta) = \dfrac{k}{k+\hat{k}}\{a^*(1) - a^*(\theta)\} = 0.935\,(1-\theta)$

Distortion: $a^{\text{HP}}(\theta) - a^*(\theta) = \dfrac{\hat{k}}{k+\hat{k}}\{a^*(1) - a^*(\theta)\} = 0.065\,(1-\theta)$



In Figure 7, the highest type firm, $\theta=1$, makes the announcement $\hat{a}=1$. Since the penalty rate $\hat{k}$ is low, all the types with $\theta<1$ mimic the highest type and announce $\hat{a}=1$. This outcome is a "high-pooling", and is denoted by the superscript, "HP."

Figure 7 shows the firm's strategy has both lying and distortion. The firm's allocation, $a^{\mathrm{HP}}(\theta)$, is the optimal tradeoff between its announcement, $\hat{a}^{\mathrm{HP}}=1$, and its cash-maximizing allocation, $a^*(\theta)=\theta$. The total difference between the firm's announcement and its cash-maximizing allocation is $1-a^*(\theta)=1-\theta$. Given the specified parameter values in Figure 7, the total difference is divided into two components: lying equal to $0.935(1-\theta)$; and distortion equal to $0.065(1-\theta)$.

From Figure 7 the average amount of lying and distortion in this pure pooling example can be calculated by taking an expectation over all types.

$$\text{Average lie} = \int_0^1 0.935(1-\theta)\,d\theta = 0.468.$$

$$\text{Average distortion} = \int_0^1 0.065(1-\theta)\,d\theta = 0.033.$$

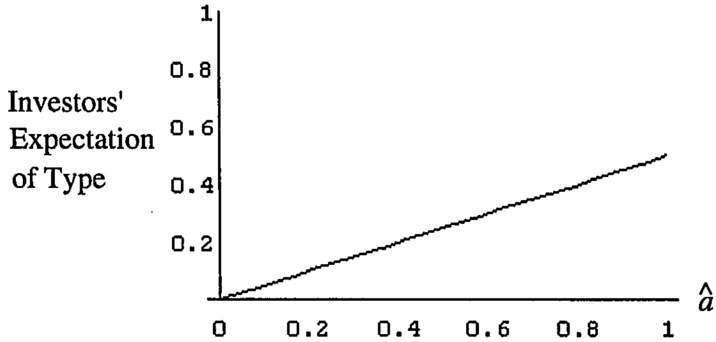
The firm strategy in the pure pooling equilibrium is sensitive to the parameter value $\hat{k}$. As $\hat{k}$ approaches zero, lying approaches $1-\theta$ and distortion approaches zero. As $\hat{k}$ increases to small positive values, the firm's optimal allocation shifts closer to the announcement $\hat{a}=1$ which decreases lying and increases distortion. Given the parameter values in Figure 7, the critical threshold is $\hat{k}=\dfrac{\beta_1 k}{3\beta_2(1+\beta_1+\beta_2)}=0.0397$. When $\hat{k}>0.0397$, mimicking the highest type becomes too costly for some low types and they deviate from pure pooling.

Figure 8. Investors' beliefs and response in pure pooling equilibrium.

Parameter values: $\beta_1 = 0.01,\quad \beta_2 = 0.04,\quad k = 0.5,\quad \hat{k} = 0.035$

Investors' posterior beliefs: $\mu(\tilde{\theta}|\hat{a}) = \tilde{\theta} \sim U(0,\hat{a})$

Investors' expectation of type: $\mathrm{E}\left[\tilde{\theta}|\hat{a}\right] = \dfrac{\hat{a}}{2}$



Investors' belief about allocation after observing announcement and inferring type:

$$a^{HP}(\theta) = 0.065\,\hat{a} + 0.935\,\theta$$

Investors' response at time 1:

$$V_1^{HP}(\hat{a}) = \mathrm{E}_{\hat{\theta}}\left[V_2(\bullet)|\hat{a}\right] = \int_0^1 V_2\left(\theta, \hat{a}, a^{HP}(\theta), V_1^{HP}(\hat{a})\right)\frac{1}{\hat{a}}\mathrm{d}\theta = 0.357 + 0.148\,\hat{a}^2$$

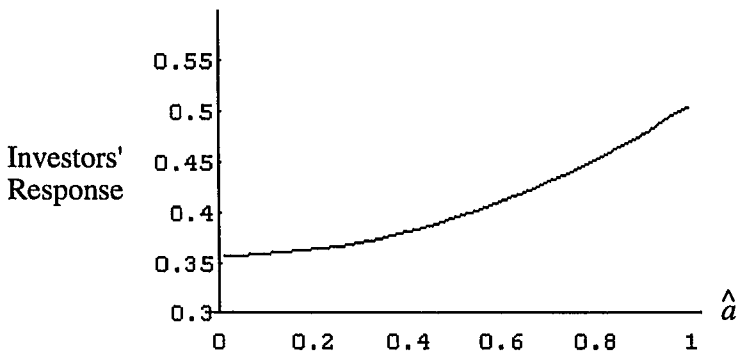


Figure 8 illustrates the investors' beliefs and response for all feasible announcements, $\hat{a} \in [0,1]$. If the firms follow the pure pooling announcement strategy illustrated in Figure 7, the equilibrium announcement set is a single announcement, $\hat{A}^e = \{\hat{a} = 1\}$. In equilibrium, the investors' prior and posterior beliefs are equal, $\mu(\tilde{\theta}) = \mu(\tilde{\theta}|\hat{a} = 1) = \tilde{\theta} \sim U(0,1)$.

In Figure 8, I exogenously specify investors' beliefs for the out-of-equilibrium announcement set, $\hat{A}^o = \{\hat{a}|0 \le \hat{a} < 1\}$. I assume if an out-of-equilibrium announcement, $\hat{a}^o$, is observed, investors believe the type is uniformly distributed on $[0, \hat{a}^o]$. Given the parameter value k=0.5, the full revelation strategy is $a^*(\theta) = \hat{a}^*(\theta) = \theta$. Suppose the out-of-equilibrium announcement, $\hat{a}^o$=0.7 is observed. A firm of type θ=0.7 would announce $\hat{a}$=0.7 in a full-revelation equilibrium. The assumed out-of-equilibrium beliefs imply the investors believe $\hat{a}^o$=0.7 would be announced by any type θ=0.7 or lower.

The first plot in Figure 8 shows how investors' beliefs about type change over the set of feasible announcements. Assumption (A-6) at the end of chapter 3 requires the investors' posterior expectation of type, $E[\tilde{\theta}|\hat{a}]$, to be a nondecreasing function of the announcement $\hat{a}$. The first plot shows that given the assumed out-of-equilibrium beliefs, the investors' expectation of type is a linear function of $\hat{a}$. If the in-equilibrium announcement $\hat{a}$=1 is observed, the investors believe the types are uniformly distributed on $[0,1]$, and the average type is 0.5.

To estimate firm value, the investors must infer both the firm's type and its allocation. I assume investors use both the observed announcement $\hat{a}$ and the inferred type θ to predict the allocation a. In Figure 8, investors believe the allocation a is a compromise between its first-best allocation $a^*(\theta) = \theta$ and its announcement $\hat{a}$. The weighting is a function of parameter $\hat{k}$. The investors' inference about the firm's allocation is consistent with the firm's in-equilibrium allocation rule shown in Figure 7, $a^{\text{HP}}(\theta) = 0.065 + 0.935\theta$.

The second plot in Figure 8 shows $V_1^{\text{HP}}(\hat{a})$, the investors' valuation response at time 1 is a quadratic function of the announcement. This valuation is a function of the investors' beliefs about type, $\mu(\tilde{\theta}|\hat{a})$, and the inferred firm allocation, $a^{\text{HP}}(\theta)$. When the firm makes the in-equilibrium announcement, $\hat{a} = 1$, the investors' response is $V_1^{\text{HP}}(1) = 0.505$. The investors' response in the first-best case, shown in Figure 4, assumes that if $\hat{a} = 1$, then the investors believe the type was $\theta = 1$ and value the firm at $V_1^*(1) = 0.833$. The investors' response to $\hat{a} = 1$ in the pure pooling equilibrium is lower than the first-best case, because the investors adjust their response for the expected amount of lying and distortion by firms.

To verify that the investors' response is in equilibrium, the following steps show that expected profits in equilibrium are zero. The investors' profits are the difference between their redemption proceeds at time 2, $V_2(\bullet)$, and the price paid for the stock at time 1, $V_1^{\text{HP}}(\hat{a})$. The expectation of $V_2(\bullet)$ is taken over the entire range of types $\theta \in [0,1]$ and assumes all firms follow the strategies, $\hat{a}^{\text{HP}}(\theta)$ and $a^{\text{HP}}(\theta)$.

$$\int_0^1 V_2\left(\theta, \hat{a}^{\mathrm{HP}}, a^{\mathrm{HP}}(\theta), V_1^{\mathrm{HP}}(\hat{a}^{\mathrm{HP}})\right)\frac{1}{\hat{a}^{\mathrm{HP}}}\mathrm{d}\theta \;-\; V_1^{\mathrm{HP}}(\hat{a}^{\mathrm{HP}})$$

$$= \int_0^1 V_2\left(\theta, 1, 0.065 + 0.935\theta, V_1^{\mathrm{HP}}(1)\right)\frac{1}{1}\mathrm{d}\theta \;-\; \{0.357 + 0.148(1)\}$$

$$= \int_0^1 \{0.324 + 0.063\theta + 0.449\theta^2\}\mathrm{d}\theta \;-\; 0.505$$

$$= 0.324 \;+\; 0.063\left(\frac{1}{2}\right) \;+\; 0.449\left(\frac{1}{3}\right) \;-\; 0.505$$

$$= 0$$

To verify that the firm's announcement rule $a^{\mathrm{HP}}(\theta)$ is a best-response to the investors' valuation function, consider a deviation by the lowest type, $\theta = 0$. The lowest type's in-equilibrium strategy is the mimicking announcement $\hat{a} = 1$ and a slightly distorted allocation of $a^{\mathrm{HP}}(0) = 0.065$. The announcement $\hat{a} = 1$ results in the investor response $V_1^{\mathrm{HP}}(1) = 0.505$ and gives the lowest type a payoff of

$$W\left(0, 1, 0.065, V_1^{\mathrm{HP}}(1)\right) = \beta_1(0.505) \;+\; \beta_2\,(0.324) = 0.0180.$$

If the lowest type deviated from the $\hat{a} = 1$ pure pooling equilibrium to its first-best strategy, $\hat{a}^*(0) = a^*(0) = 0$, then the investors' response is $V_1^{\mathrm{HP}}(0) = 0.357$ and the lowest type's payoff would be

$$W\left(0, 0, 0, V_1^{\mathrm{HP}}(0)\right) = \beta_1(0.357) \;+\; \beta_2\,(0.357) = 0.0179.$$

Deviating from the $\hat{a} = 1$ pure pooling to the first-best strategy has the following effects on the lowest type's payoff:

decrease at time 1 by $\beta_1(0.505 \;-\; 0.357) = 0.00148$, and

an increase at time 2 of $\beta_2(0.357 \;-\; 0.324) = 0.00132$.

Since the decrease at time 1 is greater than the increase at time 2, the lowest type will not deviate from pure pooling to full revelation. The proof of Proposition 4 shows there are parameter values such that no type deviates from the pure pooling announcement strategy.

Figure 9. Market return in pure pooling equilibrium.

Parameter values: $\beta_1 = 0.01$, $\beta_2 = 0.04$, $k = 0.5$, $\hat{k} = 0.035$

Only announcement observed in equilibrium is $\hat{a}^{HP} = 1$, as indicated by • in plot below.

Before observing $\hat{a}$, price is $V_0^{HP} = E_{\tilde{\theta}}\left[V_1^{HP}\left(\hat{a}^{HP}\left(\tilde{\theta}\right)\right)\right] = 0.357 + 0.148(1)^2 = 0.505$.

After observing $\hat{a}$, price is $V_1^{HP}(\hat{a}) = 0.357 + 0.148\hat{a}^2$.

Return is $R^{HP}(\hat{a}) \equiv \dfrac{V_1^{HP}(\hat{a}) - V_0^{HP}}{V_0^{HP}}$

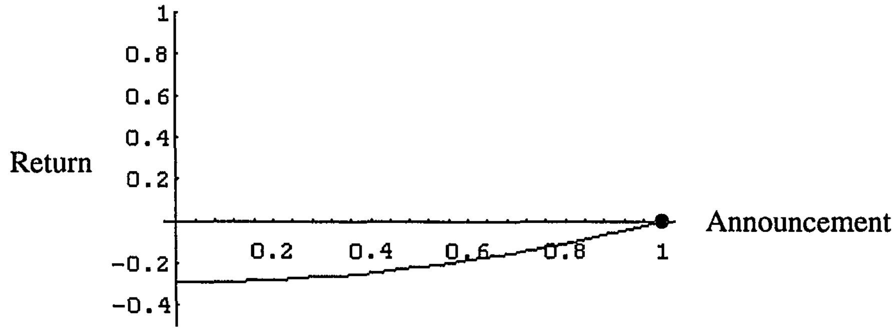


Figure 9 illustrates the market return for all feasible announcements. The only announcement observed in equilibrium is $\hat{a} = 1$. Before and after observing the announcement $\hat{a} = 1$, the investors believe the type is uniform on $[0,1]$. Thus, there is no price change when a firm announces $\hat{a} = 1$. If a firm deviates to some out-of-equilibrium announcement, $\hat{a} < 1$, then the price change is negative. If the lowest feasible announcement, $\hat{a} = 0$, is observed, the return is negative 29.3 percent.

The pure pooling equilibrium can be explained in the language of this model's economic setting. Initially, all firms have resources equally divided between two divisions, the old ESTABLISHED division and the new DEVELOPMENT division. Firm managers privately observe their prospects for DEVELOPMENT. Suppose Zero, Inc. privately observes very poor prospects in DEVELOPMENT and could maximize operating cash flow by shifting all resources to ESTABLISHED. If Zero, Inc. announces a shifting of all resources to ESTABLISHED, then Zero will be revealed as the lowest type, and the stock price will be lower than for any other firm. By announcing that it is moving all resources to DEVELOPMENT, Zero receives a higher stock price at time 1. Contrary to its announcement, Zero moves nearly all resources to ESTABLISHED. At time 2, the difference between the announcement and actual allocation is revealed. Zero is penalized at time 2, but the magnitude of the penalty at time 2 is smaller than the benefits of lying at time 1. The firm mitigates the penalty by leaving some resources at DEVELOPMENT rather than moving all resources to ESTABLISHED. The set-up of this model does not require a minimum amount of resources at either division.

4.5.1. Partial Pooling Equilibrium - Introduction

In this section the value of the penalty factor $\hat{k}$ is sufficiently large that the costs of mimicking becomes so large for low types that they choose not to mimic the highest type. The pure pooling equilibrium discussed in the previous section assumed a sufficiently low value of the penalty factor $\hat{k}$ such that all types made the same announcement, $\hat{a}^{HP}(\theta)=1$. The critical threshold is $\hat{k}=\dfrac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$.

When $\hat{k}$ exceeds the critical threshold, the pure pooling equilibrium unravels because the cost of mimicking is so high that the lowest type prefers to reveal itself rather than mimic. The number of equilibrium pools makes a discontinuous jump at the critical threshold. When $\hat{k}$ is less than the threshold, there is a pure pooling equilibrium with a single pool. When $\hat{k}$ is greater than the threshold, there is a partial pooling equilibrium with an infinite number of pools.

The discussion of the partial pooling equilibrium is divided into several subsections. Subsections 4.5.2 and 4.5.3 present the firm's strategy and the investors' beliefs, respectively. Figures in these two subsections demonstrate the partial pooling equilibrium for a particular set of exogenous parameter values. The proofs in the Appendix are not restricted to this set of parameter values. Subsection 4.5.4 analyzes the partial pooling equilibrium for the special case where the penalty rate $\hat{k}$ is infinite. Finally, subsection 4.5.5 presents the sensitivity of the results to changes in the exogenous parameters.

Proposition 5 proposes a partial pooling equilibrium and describes its major characteristics. The figures in the following sections illustrate some of these characteristics. The Appendix contains the proof that the proposed equilibrium satisfies the equilibrium requirements specified in section 3.6. The investors' beliefs that support this equilibrium are discussed in section 4.5.3 and the Appendix. The characteristics of the interval boundaries are discussed in section 4.5.2 and in Lemma 2 and Lemma 3 in the Appendix.

Proposition 5. If the firm privately observes its type, and the penalty factor $\hat{k}$ is a sufficiently large finite value, such that $\frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)} < \hat{k} < +\infty$, then a partial pooling equilibrium occurs. This partial pooling equilibrium is characterized by:

(i) Almost all firms lie.

(ii) Almost all firms distort.

(iii) There are an infinite number of intervals.

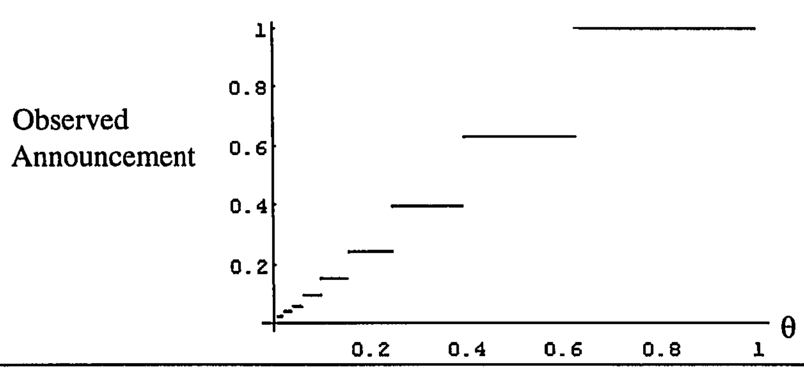
Proof: See Appendix at page 150.

4.5.2. Partial Pooling Equilibrium - Firm's Strategy

When $\hat{k}$ is sufficiently large there exists a partial pooling equilibrium with a countably infinite number of pools. This partial pooling is characterized by a parameter *r*, such that $0 < r < 1$, and the infinite decreasing geometric series $\{r^0, r^1, r^2, \ldots r^j, \ldots\}$. Since $0 < r < 1$, this series begins with $r^0 = 1$ and decreases. This geometric series partitions the range of types into a countably infinite set of intervals, $\{\ldots, (r^{j+1}, r^j], (r^j, r^{j-1}], \ldots, (r^2, r], (r, 1]\}$. Intervals are indexed by the exponent j, for the integers j=0,1,2,... Interval j is $\left(r^{j+1}, r^{j}\right]$ and has length $r^{j} - r^{j+1}$. Parameter r is the ratio of the length of interval j+1 to the next-higher interval j.

Figure 10 presents the firm's announcement strategy in a partial pooling equilibrium where r=0.63. The first three members of the geometric series, rounded to two significant digits, are $r^0 = 1$, $r^1 = 0.63$, and $r^2 = 0.39$. Intervals are indexed from right to left. All firms in the first interval, $\theta \in (0.63, 1]$, announce $\hat{a} = 1$. All firms in the second interval, $\theta \in (0.39, 0.63]$, announce $\hat{a} = 0.63$. The highest type within each interval makes its first-best announcement, $\hat{a}^*(\theta) = a^*(\theta)$. All other types within an interval mimic the announcement of the interval's highest type.

Figure 10. Observed announcements in a partial pooling with $\hat{k}$=1 and r=0.63.
Parameter values: $\beta_1 = 0.01$, $\beta_2 = 0.04$, $k = 0.5$, $\hat{k} = 1$, $r = 0.63$
In-equilibrium announcement set: $\hat{A}^e = \{\ldots, .63^j, \ldots, 0.25, 0.39, 0.63, 1\}$



Observe the intervals in Figure 10 become shorter as type decreases from the maximum type, $\theta = 1$, toward the minimum type, $\theta = 0$. The first interval,

$(0.63, 1]$, has length $1 - r = 0.37$. The second interval, $(0.39, 0.63]$, has length $r - r^2 = r(1-r) = 0.24$. Each interval is r=0.63 times as long as the interval to its right. As type approaches zero, the intervals become infinitesimally small, but are not single points.

The decreasing length of the intervals occurs because the benefits of mimicking are larger for the higher announcement intervals. This result is a consequence of assuming a quadratic production function, $Z(\bullet)$, as specified at (A-2) in section 3.3. In the first-best case, Figure 4 illustrates that if all firms follow the cash-maximizing strategy $\hat{a}^*(\theta) = a^*(\theta) = \theta$, then the increase in the investors' response, $V_1{}^*(\hat{a})$, accelerates as the announcement increases. For example, the price difference between $\hat{a} = 1$ and $\hat{a} = 0.5$ is $V_1{}^*(1) - V_1{}^*(0.5) =$ $0.833 - 0.476 = 0.357$, and that is greater than the price difference between $\hat{a} = 0.5$ and $\hat{a} = 0$ which is $V_1{}^*(0.5) - V_1{}^*(0) =$ $0.476 - 0.357 = 0.119$. The investors' price response to equilibrium announcements in the partial pooling increases quadratically, but not as steeply as in the first-best case.

The lack of an upper bound to the number of intervals in the partial pooling is a consequence of assuming continuous types. If the number of types were discrete, then the upper limit for the number of announcement intervals would be the number of discrete types. However, there are an uncountable number of types when the feasible types are continuous. Step 7 of the proof of Proposition 5 in the Appendix proves the number of intervals is infinite.

The partial pooling equilibrium in this section shares some characteristics with the partition equilibria described in Crawford and Sobel [1982] and Newman and Sansing [1993]. In all three papers types on the partition boundaries are indifferent between two adjacent intervals. The technical condition for the boundary indifference condition in my model is given by Lemma 2.

Lemma 2. Given a parameter value r such that $0 < r < 1$ and r is a solution to

$$\Omega(r) = \beta_1(1+r)(1+r+r^2) + 6\hat{k}\{\beta_1 r(1+r) - \beta_2(1+\beta_1+\beta_2)(1-r)\} = 0, \quad \textbf{(I)}$$

and firm's allocation $a^{\text{TPP}}(\theta) = \dfrac{\theta + 2\hat{k}r^{j}}{1+2\hat{k}}$ for $\theta \in (r^{j+1}, r^{j}]$,

and $V_1^{\text{TPP}}(\hat{a}) = \dfrac{1}{1+\beta_1+\beta_2}\displaystyle\int_{r^{j}}^{r^{j-1}} Z(\theta, \hat{a}, a^{\text{TPP}}(\theta))\frac{1}{r^{j}-r^{j+1}}d\theta$ for $\hat{a} \in [r^{j}, r^{j-1})$;

then any boundary type $\theta = r^{j}$ where j=1,2,..., is indifferent between announcing $\hat{a}*(r^{j})$ or $\hat{a}*(r^{j-1})$.

Proof: See Appendix at page 148.

Lemma 2 implies that if interval ratio r exists, then it satisfies the indifference condition **(I)**. Lemma 3 goes further and says that given the parameter conditions I assumed, there always exists an r that satisfies the indifference condition.

Lemma 3. If $\beta_1 > 0$, $\beta_2 > 0$, $k = \dfrac{1}{2}$, $\hat{k} > \dfrac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$, then there exists a solution r such that satisfies **(I)**.

Proof: See Appendix at page 149.

When the exogenous parameters are β_1=0.01, β_2=0.04, k=0.5, and $\hat{k}$=1 as in Figure 10; the indifference equation (**I**) from Lemma 2 is:

$$-0.242 + 0.332r + 0.080r^2 + 0.01r^3 = 0$$

This equation has one real-valued solution, r=0.626822≈0.63.

In Figure 10 the type θ=r=0.63 is indifferent between its first-best announcement $\hat{a} = 0.63$ or the mimicking announcement $\hat{a} = 1$. Similarly, the type θ=r^2=0.39 is on the boundary between the second and third intervals, and is indifferent between its first-best announcement $\hat{a} = 0.39$ or the mimicking announcement $\hat{a} = 0.63$. Thus the boundary type, $\theta = r^j$, is indifferent between its first-best announcement, $\hat{a}*(r^j)$, and some other announcement. To avoid allowing boundary types to randomize between two announcements, I assume that the boundary type chooses its first-best announcement.

Unlike Crawford-Sobel and Newman-Sansing, the firms in my model do not randomize among all the feasible announcements within an interval as illustrated in Figure 1a in section 2.6. In the former models, the firm's announcement is "cheap talk" that may influence the beliefs of other players, but does *not* directly enter the firm's payoff function. However, in my model, the announcement $\hat{a}$ influences the investors' beliefs and *does* directly enter the firm's payoff through the penalty term $-\hat{k}(\hat{a}-a)^2$.

Figure 11. Firm's strategy in a partial pooling with $\hat{k}$=1 and r=0.63.

Parameter values: $\beta_1 = 0.01,\ \ \beta_2 = 0.04,\ \ k = 0.5,\ \hat{k} = 1,\ r = 0.63$

$$\hat{a}^{rpp}(\theta) = r^j = 0.63^j \quad \text{for} \quad \theta \in \left(0.63^{j+1},\ 0.63^j\right] \quad \text{and} \quad j = 0,1,2,\ldots$$

$$a^{rpp}(\theta) = \frac{\theta + 2\hat{k}r^j}{1 + 2\hat{k}} = 0.333\theta \ +\ 0.667\,(0.63)^j \quad \text{for} \quad \theta \in \left(r^{j+1},\ r^j\right]$$

Lying: $\hat{a}^{rpp}(\theta) - a^{rpp}(\theta) = \frac{1}{1+2\hat{k}}\left(r^j - \theta\right) = 0.333\left\{(0.63)^j - \theta\right\}$ for $\theta \in \left(r^{j+1}, r^j\right]$

Distortion: $a^{rpp}(\theta) - a^*(\theta) = \frac{2\hat{k}}{1+2\hat{k}}\left(r^j - \theta\right) = 0.667\left\{(0.63)^j - \theta\right\}$ for $\theta \in \left(r^{j+1}, r^j\right]$

Plot of announcement and allocation by firm type.

Horizontal lines indicate announcement function $\hat{a}^{rpp}(\theta)$.

Positively sloped lines indicate allocation function $a^{rpp}(\theta)$.

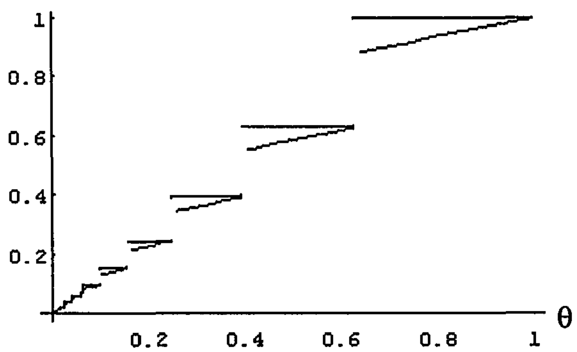


Figure 11 illustrates the firm's announcement and allocation strategies in a partial pooling equilibrium. The strategy functions in Figure 11 are denoted by the superscript "rpp" to indicate the value r characterizes this partial pooling equilibrium. The proof of Proposition 5 in the Appendix proves that this is an equilibrium by showing that no firm deviates from the announcement and allocation strategies, $\hat{a}^{\text{TPP}}(\theta)$ and $a^{\text{TPP}}(\theta)$.

The highest interval in Figure 11 is $(0.63, 1]$. All types in the interval $(0.63, 1]$ announce $\hat{a} = 1$ and make the resource allocation $a^{\text{TPP}}(\theta) = 0.333(\theta + 2)$. In the first-best outcome the firm's strategy is $\{\hat{a} = \theta, a = \theta\}$, which is defined as no lying and no distortion. Figure 11 shows that almost all types do not chose the first-best strategy. The boundary types, $\theta \in \{\ldots, 0.63^j, \ldots 0.25, 0.39, 0.63, 1\}$, do chose the first-best. The types not on the interval boundaries mimic the announcement of the highest type in the interval. Thus, all types not on the interval boundaries have a positive amount of lying.

The allocation strategy $a^{\text{TPP}}(\theta) = 0.333(\theta + 2)$ for firms in the interval $(0.63, 1]$ is the optimal tradeoff between lying and distorting. Given the restriction $k = 0.5$ the firm could achieve zero distortion with the allocation $a = a^*(\theta) = \theta$. The allocation strategy $a^{\text{TPP}}(\theta) = 0.333(\theta + 2)$ for firms in the interval $(0.63, 1]$ is the optimal tradeoff between lying and distorting. With the parameters selected for this example, for firms in the interval $(0.63, 1]$, the firm's compensation-maximizing allocation, $a^{\text{TPP}}(\theta)$, is one-third of the distance between its announcement $\hat{a} = 1$ and its cash-maximizing allocation $a^*(\theta)$. The difference between $a^{\text{TPP}}(\theta)$ and the cash-maximizing allocation is distortion, $a^{\text{TPP}}(\theta) - a^*(\theta) = 0.667(1 - \theta)$.

In general, the optimal amount of lying and distortion is a linear function of the exogenous parameter $\hat{k}$. As $\hat{k}$ increases, lying becomes more costly; the optimal amount of lying decreases; and distortion increases. The ratio of optimal lying to optimal distortion for a particular interval equals the ratio $\frac{k}{\hat{k}}$. Thus, in the example where k=0.5 and $\hat{k}=1$, the optimal amount of lying is half the optimal amount of distortion.

The comparison of equilibria is facilitated by calculating the average amount of lying and distortion for this partial pooling example where $\hat{k}=1$. The average lie is the expected difference between the announcement and allocation, $\int_0^1 \{\hat{a}^{rpp}(\theta)-a^{rpp}(\theta)\}\,d\theta = 0.038$. The average distortion is the expected difference between the allocation chosen in equilibrium and the cash-maximizing allocation, $\int_0^1 \{a^{rpp}(\theta)-a^*(\theta)\}\,d\theta = 0.076$. The average lie is half the average distortion, because the ratio $\frac{k}{\hat{k}}$ is 0.5 in this example. Section 4.5.4 shows that when $\hat{k}$ is increased to positive infinity, the average lie is zero; and the average distortion, 0.084.

The relationship between distortion and lying in the partial pooling equilibrium has some policy implications. Increasing the effective penalty for Rule 10b-5 violations is represented in this model by an increase in the penalty rate $\hat{k}$. The results of the analysis in the partial pooling equilibrium in section 4.5.4 shows that as $\hat{k}$ increases, the amount of lying decreases and distortion increases.

This result occurs because the model assumes firms are sued for false announcements; but firm managers are not sued if they make an operating decision that does not maximize cash flow. In my model at time 1 the investors are uncertain about the productivity θ, but at time 2 they know a, $\hat{a}$, and $Z(\theta,\hat{a},a)$. At time 2 the investors could invert $Z(\theta,\hat{a},a)$ and determine θ and the cash-maximizing allocation $a^*(\theta)=\theta$ with certainty. Therefore, at time 2, the investors could infer the amount of distortion. However, at time 2 the model does not allow investors to directly assess a personal penalty on managers who distorted. Distortion does reduce the firm manager's compensation, because distortion does is an opportunity cost to both the manager and the investors. With a distorted allocation the cash available for distribution, $Z(\theta,\hat{a},a)$, is less than it would be with the cash-maximizing allocation $a^*(\theta)=\theta$. In equilibrium, the investors' average profits at time 2 are zero, because they adjust their pricing response at time 1 for the expected amount of distortion.

I recognize that in the real world investors and regulatory agencies can penalize managers for either false disclosures (lying) or operating decisions that do not maximize resource efficiency (distortion). I model only one penalty for lying: a Rule 10b-5 penalty that reduces firm value for both the manager and the investors. In the real world managers who do not manage resources efficiently can be sued personally or fired for failing to carry out their fiduciary duty to the stockholders. A possible extension of the model set-up is to add a personal penalty on managers who distort.

4.5.3. Partial Pooling Equilibrium - Investors' Beliefs

Each partition in this partial pooling example is characterized by a discrete announcement, $\hat{a} \in \hat{A}^e = \{0.63^j \text{ for } j = 0,1,2,...\}$. In this example the out-of-equilibrium announcement set is $\hat{A}^o = \{\hat{a} | \hat{a} \in [0,1] \text{ and } \hat{a} \neq 0.63^j \text{ for } j = 0,1,2,3,...\}$. This aspect of my partial pooling is unlike the partition equilibrium of Newman-Sansing where all feasible announcements are observed in equilibrium.

For the partial pooling equilibrium I assume if an out-of-equilibrium announcement strictly between the in-equilibrium announcement of two adjacent intervals is observed, then the investors believe the firm is from the lower interval. These beliefs are illustrated in Figure 12. For example, if the in-equilibrium announcement $\hat{a} = 1$ is observed, the investors believe the firm's type is contained in the highest interval, $(0.63, 1]$, and the mean of this interval is 0.815. If an announcement strictly between 0.63 and 1 is observed, the investors believe the firm's type is contained in the interval $(0.39, 0.63]$, and the mean of this interval is 0.51. If the in-equilibrium announcement $\hat{a} = 0.63$ is observed, the investors believe the firm's type is contained in the interval $(0.39, 0.63]$ with a mean of 0.51.

Figure 12. Investors' beliefs in a partial pooling with $\hat{k}=1$ and $r=0.63$.

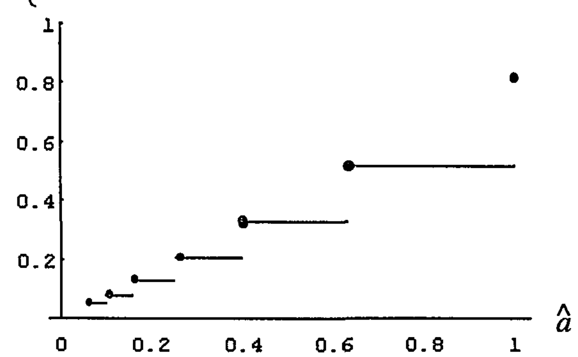
Parameter values: $\beta_1 = 0.01,\ \beta_2 = 0.04,\ k = 0.5,\ \hat{k} = 1,\ r = 0.63$

Investors' beliefs: $$\mu\left(\tilde{\theta}\middle|\hat{a}\right) = \begin{cases} \tilde{\theta} \sim U(0.63,1) & \text{if } \hat{a} = 1 \\ \tilde{\theta} \sim U(.63^2,.63) & \text{if } \hat{a} \in [.63,1) \\ \tilde{\theta} \sim U(.63^{j+1},.63^j) & \text{if } \hat{a} \in [.63^j,.63^{j-1}) \text{ and } j = 2,3,\ldots \\ \{\theta = 0\} & \text{if } \hat{a} = 0 \end{cases}$$

Investors' expectation of type:

$$E\left[\tilde{\theta}\middle|\hat{a}\right] = \begin{cases} (0.63+1)/2 & \text{if } \hat{a} = 1 \\ (0.63^{j+1} + 0.63^j)/2 & \text{if } \hat{a} \in [0.63^j,\ 0.63^{j-1}) \text{ and } j = 1,2,3,\ldots \\ 0 & \text{if } \hat{a} = 0 \end{cases}$$

Investors' Expectation of Type



• indicates in-equilibrium announcement, $\hat{a} \in \hat{A}^e = \{\ldots 0.63^j, \ldots, 0.25, 0.39, 0.63, 1\}$

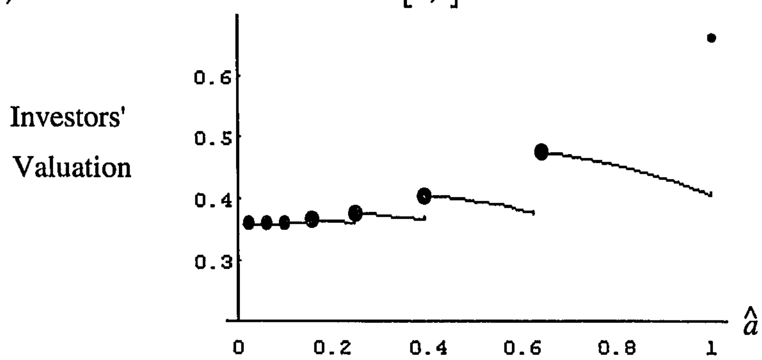
Figure 12 shows how investors' expectations about type change over the set of feasible announcements. Assumption (A-6) at the end of chapter 3 requires that investors' posterior expectation of type, $E\left[\tilde{\theta}\middle|\hat{a}\right]$, is a nondecreasing function of the announcement $\hat{a}$. Figure 12 shows that given the assumed out-of-equilibrium beliefs, the investors' expectation weakly increases in $\hat{a}$.

Figure 13. Investors' pricing response in a partial pooling with $\hat{k}$=1 and r=0.63.

Parameter values: $\beta_1 = 0.01, \ \ \beta_2 = 0.04, \ \ k = 0.5, \ \hat{k} = 1, \ r = 0.63$

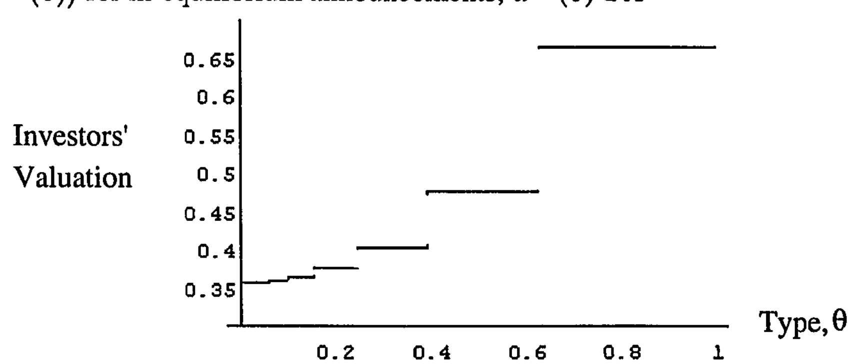
Investors' beliefs: $\mu(\tilde{\theta}|\hat{a})$ as in Figure 12

$V_1^{\text{rpp}}(\hat{a})$ as a function of all feasible $\hat{a} \in [0,1]$



• indicates in-equilibrium announcement, $\hat{a} \in \hat{A}^e = \{...0.63^j, ..., 0.25, 0.39, 0.63, 1\}$

$V_1^{\text{rpp}}(\hat{a}^{\text{rpp}}(\theta))$ for in-equilibrium announcements, $\hat{a}^{\text{rpp}}(\theta) \in \hat{A}^e$



The first plot in Figure 13 illustrates how the investors' valuation response, $V_1^{\text{rpp}}(\hat{a})$, changes over the feasible set of announcements. The function $V_1^{\text{rpp}}(\hat{a})$ is not monotonic for announcements in the out-of-equilibrium set, $\hat{A}^o$. For example, consider the interval of out-of-equilibrium announcements strictly between 0.63 and 1. If any announcement in this interval is observed, then investors believe the firm's type is contained in the interval $(0.39, 0.63]$ with a mean value of 0.51. As the announcement $\hat{a}$ increases from the 0.63 toward 1, the expected difference between 0.513 and $\hat{a}$ increases. If the investors believe the gap between type and announcement is increasing, then they believe the firm will have a larger penalty for lying. Although increasing $\hat{a}$ from 0.63 to 1 does not change investors expectation about type, it does increase their expectations about the amount of lying. Therefore, the valuation $V_1^{mp}(\hat{a})$ strictly decreases as $\hat{a}$ increases from 0.63 to 1.

The second plot in Figure 13 shows that over the discrete set of in-equilibrium announcements, $\hat{A}^e$, the function $V_1^{mp}(\hat{a})$ monotonically increases. For example, the four highest in-equilibrium announcements, 0.25, 0.39, 0.63, and 1, have values of $V_1^{mp}(\hat{a})$ equal to 0.38, 0.41, 0.48, and 0.66 respectively.

Observe that the difference between the in-equilibrium announcements accelerates as the type and announcements increase. As the announcements increase, there is a bigger price change from mimicking the higher announcement. This provides more incentive for the higher types to mimic, and the higher intervals are longer than the lower intervals. As announcements decrease, the difference in prices declines; the incentive to mimic is reduced; and the interval length decreases.

Figure 14. Market return in a partial pooling with $\hat{k}$=1 and r=0.63.

Parameter values: $\beta_1 = 0.01$, $\beta_2 = 0.04$, $k = 0.5$, $\hat{k} = 1$

Before observing $\hat{a}$, price is $V_0^{\text{rpp}} = \mathrm{E}_{\tilde{\theta}}\left[V_1^{\text{rpp}}\left(\hat{a}^{\text{rpp}}(\tilde{\theta})\right)\right] = 0.509$.

After observing $\hat{a}$, price is $V_1^{\text{rpp}}(\hat{a})$ as shown in Figure 13.

Return is $R^{\text{rpp}}(\hat{a}) \equiv \dfrac{V_1^{\text{rpp}}(\hat{a}) - V_0^{\text{rpp}}}{V_0^{\text{rpp}}}$

In-equilibrium announcements indicated by • in this plot.

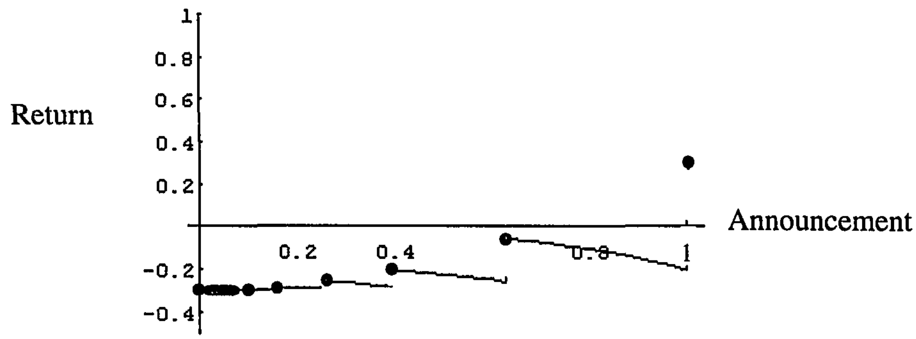


Figure 14 illustrates the market return reaction in the partial pooling equilibrium with $\hat{k} = 1$. Figure 14 presents the investors' response as a return relative to the initial price, V_0^{rpp}, whereas Figure 13 presents the posterior prices, $V_1^{\text{rpp}}(\hat{a})$, on an absolute price scale. The dark dots in Figure 14 indicate the market return to in-equilibrium announcements and the curved lines indicate the returns to out-of-equilibrium announcements. The announcement $\hat{a} = 1$ has a positive return of 30.4 percent. The lowest announcement $\hat{a} = 0$ has a negative return of 29.8 percent. The price return is much more sensitive to announcement changes when announcements are high than when announcements are low.

In Figure 14 nearly all the in-equilibrium restructuring announcements result in a negative market return. The return is positive only for the highest in-equilibrium announcement, $\hat{a} = 1$. The average type is θ=0.5. Only firms with sufficiently high types, θ>0.63, announce $\hat{a} = 1$ and receive a positive return. All other types, $\theta \leq 0.63$, make an inferior announcement and receive a negative return. For example a firm with the average type, $\theta = 0.5$, announces $\hat{a} = 0.63$, and receives a return of negative 5.5 percent.

The return behavior in Figure 14 is consistent with the empirical observations that motivated this dissertation in chapter 1. The market return is positive for some restructuring announcements and negative for others. Firms with the very best prospects announce restructurings where they abandon old established segments and move toward developing segments, and receive a favorable return. Firms with average prospects announce partial restructurings toward the developing segments, but receive a slightly negative return. Firms with the worst prospects announce a retreat to the old established segments, and receive the most negative reaction. The median market return is negative, but there is a large variance in returns.

4.5.4. Partial Pooling Equilibrium - When Penalty Is Infinite

When the penalty rate $\hat{k}$ approaches infinity, the cost of lying becomes so large that no firm lies. In this model no lying means all firms chose an allocation equal to their announcement. However, Corollary 3.2 shows an infinite penalty on lying will not result in the first-best outcome. Proposition 6 states that a partial pooling equilibrium exists when the penalty rate $\hat{k}$ approaches infinity.

Proposition 6. If the firm privately observes its type and the penalty factor $\hat{k}$ is infinitely large, then a partial pooling equilibrium occurs. This partial pooling equilibrium is characterized by:

(i) No firm lies.

(ii) Almost all firms distort.

(iii) There are an infinite number of intervals.

Proof: See Appendix at page 163.

The partial pooling equilibrium in Proposition 6 with an infinite $\hat{k}$ is similar to the partial pooling in Proposition 5 with a large finite $\hat{k}$. Both equilibria involve a partition on the range of types with a countably infinite set of intervals, $\{\ldots,(r^{j+1},r^{j}],\ (r^{j},r^{j-1}],\ldots,(r^{2},r],\ (r,1]\}$. Interior boundary types, $\theta \in \{\ldots,r^{3},r^{2},r\}$, are indifferent between adjacent intervals. The parameter r is a function of the other parameters and reaches an upper bound when $\hat{k}$ is infinite. The firm's strategy has some distortion when $\hat{k}$ is infinite, but no lying. When $\hat{k}$ is finite, the firm's strategy has both distortion and lying.

Figure 15. Firm's strategy in a partial pooling with $\hat{k}=\infty$ and r=0.71.

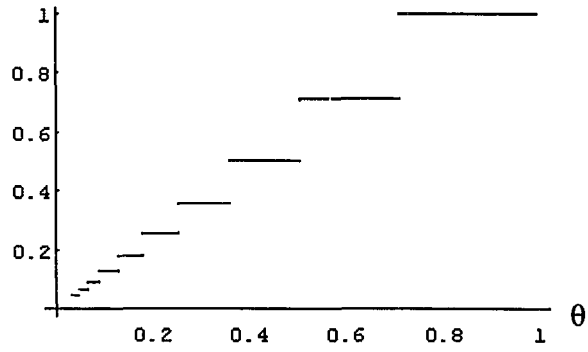
Parameter values: $\beta_1=0.01,\ \beta_2=0.04,\ k=0.5,\ \hat{k}=\infty,\ r=0.71$

$\hat{a}^{r\infty}(\theta)=r^j=0.71^j$ for $\theta\in\left(0.71^{j+1},\ 0.71^j\right]$ and $j=0,1,2,\ldots$

$a^{r\infty}(\theta)=\hat{a}^{r\infty}(\theta)$

Plot of announcement and allocation by firm type.

Horizontal lines indicate announcement or allocation, $a^{r\infty}(\theta)=\hat{a}^{r\infty}(\theta)$



The proof of Proposition 6 contains an indifference equation that can be solved for r. Given the exogenous parameters are β_1=0.01, β_2=0.04, k=0.5, and $\hat{k}$=∞; the indifference equation is:

$$-0.042\ +\ 0.052r\ +\ 0.01r^2=0 \text{ subject to } 0\le r\le 1$$

This equation has one feasible solution, r=0.710589≈0.71. Figure 15 illustrates the partial pooling equilibrium in this case. The strategies are denoted by the superscript "$r\infty$" to indicate a partial pooling with $\hat{k}$=∞.

Comparison of the firm's strategy in Figure 11 and Figure 15 shows the effect of changing from $\hat{k}=1$ to $\hat{k}=\infty$. With $\hat{k}=1$, some lying occurs and the graph of the announcement and allocation functions are different. When $\hat{k}$ approaches infinity, the announcement and allocation functions have the same graph, and that implies lying is eliminated. The ratio r increases from r=0.63 when $\hat{k}=1$ to r=0.71 when $\hat{k}=\infty$. When r=0.63, the range of types between 0.25 and 1 is partitioned into three intervals. When r=0.71, the range of types between 0.25 and 1 is partitioned into four intervals. Thus, increasing $\hat{k}$ results in an increase of the fineness of the equilibrium partition. Thus, the investors have a less noisy revelation of the firm's type as $\hat{k}$ increases.

As $\hat{k}$ approaches infinity, lying is eliminated, but distortion remains. The announcement truthfully reveals the allocation, but most firms do not select the $a^*(\theta)$ allocation that maximizes the firm's cash. For most firms, the benefits of mimicking a higher type and increasing the stock price at time 1 is greater than the cost of reducing cash flow at time 2. The exceptions are the interior boundary types, which are indifferent between announcements of adjacent intervals. Given the parameters k=0.5 and $\hat{k}=\infty$, the amount of distortion for a particular type in Figure 15 is $a^{r\infty}(\theta)-a^*(\theta)=a^{r\infty}(\theta)-\theta$. The average amount of distortion is

$$\int_0^1 \{a^{r\infty}(\theta)-a^*(\theta)\}d\theta = 0.084.$$

Figure 16 illustrates the market return reaction in the partial pooling equilibrium with $\hat{k}=\infty$. The announcement $\hat{a}=1$ has a positive return of 36.4 percent. The lowest announcement $\hat{a}=0$ has a negative return of 29.9 percent. Comparing Figure 16 where r=0.71 to Figure 14 where r=0.63 shows the rate of increase in returns is steeper when r increases. When r increases, intervals become smaller, and investors can make more precise estimate of future cash flow. This implies increasing the reporting penalty $\hat{k}$ increases the disclosure of the allocation. However, increasing the reporting penalty does not necessarily motivate the firm to eliminate distortion, i. e., make the allocation that maximizes available cash.

Figure 16. Market return in partial pooling with $\hat{k} = \infty$ and r=0.71.

Parameter values: $\beta_1 = 0.01$, $\beta_2 = 0.04$, $k = 0.5$, $\hat{k} = \infty$

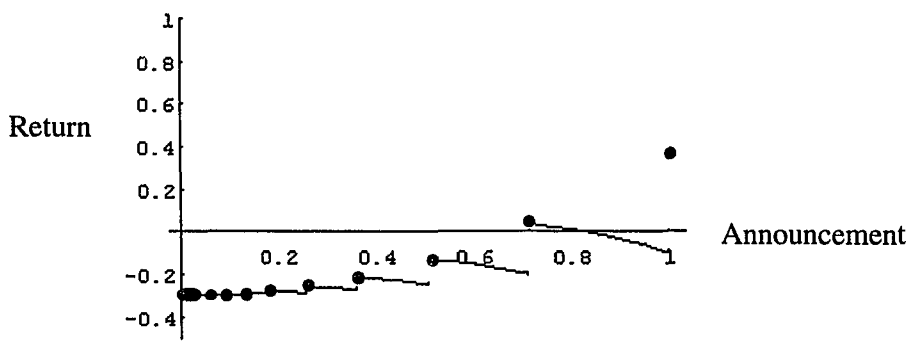
Before observing $\hat{a}$, price is $V_0^{r\infty} = \mathrm{E}_{\tilde{\theta}}\left[V_1^{r\infty}\left(\hat{a}^{r\infty}(\tilde{\theta})\right)\right] = 0.510.$

Investors' beliefs: $\mu(\tilde{\theta}|\hat{a}) = \begin{cases} \tilde{\theta} \sim U(0.71,1) & \text{if } \hat{a} = 1 \\ \tilde{\theta} \sim U(.71^{j+1},.71^{j}) & \text{if } \hat{a} \in [.71^{j},.71^{j-1}) \text{ and } j = 1,2,.. \\ \{\theta = 0\} & \text{if } \hat{a} = 0 \end{cases}$

After observing $\hat{a}$, price is $V_1^{r\infty}(\hat{a}) = \frac{1}{1.05}\mathrm{E}_{\tilde{\theta}}\left[Z\left(\tilde{\theta}, \hat{a}^{r\infty}(\tilde{\theta}), a^{r\infty}(\tilde{\theta})\right)\middle|\hat{a}, \mu(\tilde{\theta}|\hat{a})\right]$

Return is $R^{r\infty}(\hat{a}) \equiv \dfrac{V_1^{r\infty}(\hat{a}) - V_0^{r\infty}}{V_0^{r\infty}}$

In-equilibrium announcements indicated by • in this plot.



4.5.5. Partial Pooling Equilibrium - Sensitivity to Exogenous Parameters

The parameter r in the partial pooling equilibrium is a measure of the fineness of the information partition observed by the investors. An increase in r represents more complete information disclosure (less noise) to investors. Consider the interval of types $(0.66, 1]$. If r=0.66, all types in $(0.66, 1]$ are in one interval and announce $\hat{a}=1$. If r=0.9, then $(0.66, 1]$ is divided into four intervals with a distinct announcement for each interval: $(0.66, 0.73]$, $(0.73, 0.81]$, $(0.81, 0.9]$, and $(0.9, 1]$. Recall that the length of interval j is $r^j - r^{j+1}$. As r approaches one, each interval approaches a single point. If r equals one, then each type makes a unique announcement and the investors can invert the announcement to fully reveal the type and allocation. For any r less than one, some distortion or lying occurs.

The value of interval ratio r in the partial pooling equilibrium is an endogenous result of the exogenous parameters. The remainder of this section considers the sensitivity of parameter r to changes in the exogenous parameters $\hat{k}$, β_2, and β_1.

In the partial pooling equilibrium, the value of r is a solution to the indifference condition (I) in Lemma 2. (I) requires $0 < r < 1$ and r is a solution to $\Omega(r) = \beta_1(1+r)(1+r+r^2) + 6\hat{k}\{\beta_1 r(1+r) - \beta_2(1+\beta_1+\beta_2)(1-r)\} = 0$. The equation can be rearranged as a cubic equation and solved as follows.

$$\Omega(r) = \gamma_0 + \gamma_1 r + \gamma_2 r^2 = 0$$

where $\gamma_0 \equiv 1 - 6\hat{k}\frac{\beta_2}{\beta_1}(1+\beta_1+\beta_2)$

$$\gamma_1 \equiv 2 + 6\hat{k}\left\{1 + \frac{\beta_2}{\beta_1}\left(1 + \beta_1 + \beta_2\right)\right\}$$

$$\gamma_2 \equiv 2\left(1 + 3\hat{k}\right)$$

Define $\phi_1 \equiv \gamma_1 - \frac{\gamma_2^{\ 2}}{3}$ and $\phi_2 \equiv \frac{2\gamma_2^{\ 2}}{27} - 3\gamma_1\gamma_2 + \gamma_0$.

Define $A \equiv \left\{-\frac{\phi_2}{2} + \left(\frac{\phi_2^{\ 2}}{4} + \frac{\phi_1^{\ 3}}{27}\right)^{1/2}\right\}^{1/3}$ and $B \equiv \left\{-\frac{\phi_2}{2} - \left(\frac{\phi_2^{\ 2}}{4} + \frac{\phi_1^{\ 3}}{27}\right)^{1/2}\right\}^{1/3}$.

The three solutions to the cubic equation are:

$$r_1 = A + B$$

$$r_2 = -\frac{1}{2}(A + B) + \frac{i\sqrt{3}}{2}(A - B)$$

$$r_3 = -\frac{1}{2}(A + B) - \frac{i\sqrt{3}}{2}(A - B)$$

When $\frac{\phi_2^{\ 2}}{4} + \frac{\phi_1^{\ 3}}{27} > 0$, then r_1 is real-valued; and r_2 and r_3 are imaginary. The solution illustrated in Figures 17, 18, and 19 are plots of r_1 for various parameter values. The first derivatives exist, but their formulas are too complex to present in the text.

Intuitively, an exogenous increase in the penalty rate for lying should reduce the amount of lying and improve disclosure. In this model's partial pooling equilibrium, a change in the penalty rate $\hat{k}$ reduces noise for two reasons: less lying within an interval and a finer information partition.

The amount of lying within an interval is $\hat{a}^{TPP}(\theta) - a^{TPP}(\theta) = \frac{k}{k+\hat{k}}(r^j - \theta)$ for $\theta \in (r^{j+1}, r^j]$. An increase in $\hat{k}$ increases the denominator, which reduces the amount of lying. As $\hat{k}$ increases, within each interval the firm's allocation $a^{TPP}(\theta)$ shifts closer to its announcement $\hat{a}^{TPP}(\theta)$.

Figure 17. Sensitivity of *r* to $\hat{k}$

Parameter values: $\beta_1 = 0.01$, $\beta_2 = 0.04$, $k = 0.5$

$$r(\hat{k}) = \underset{r \in [0,1]}{\text{Solve}} \left\{ .01 + .02r + .02r^2 + .01r^3 - 6\hat{k}(0.042 - .052r - .01r^2) = 0 \right\}$$

Plot of *r* as a function of $\hat{k}$.

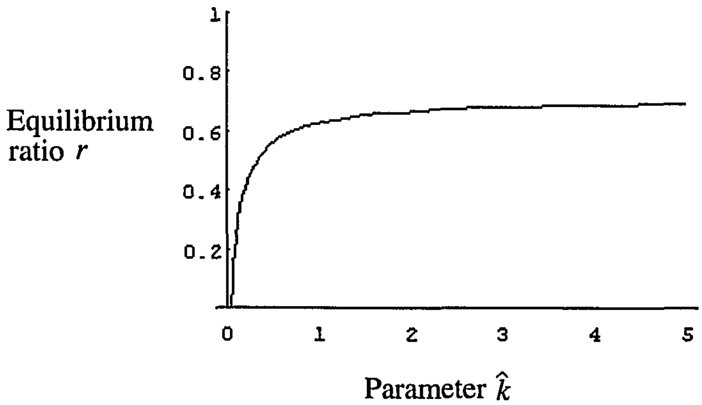


Figure 17 illustrates the sensitivity of *r* to an increase in penalty factor $\hat{k}$ while holding the values of other exogenous parameters constant. If $\hat{k}$ is sufficiently small, such that $\hat{k} \le \frac{\beta_1 k}{3\beta_2(1+\beta_1+\beta_2)} = 0.0397$, then $r \le 0$ and the partial pooling equilibrium does not exist. When $\hat{k}$ is below the threshold value, all types including the lowest type, $\theta = 0$, mimic the highest type and a pure pooling with a single interval occurs. When $\hat{k}$=1 as in Figure 11, the equilibrium *r* is 0.63.

As $\hat{k}$ increases above 1, the value of r increases more slowly. As $\hat{k}$ approaches infinity, the indifference equation approaches $-0.042 + .052r + .01r^2 = 0$. In section 4.5.4 the solution to this problem with $\hat{k} = \infty$ is identified as $r = 0.71$. Thus, in Figure 15 r approaches an upper limit of 0.71 as $\hat{k}$ increases.

An increase in parameter value β_2 implies the firm manager's compensation function places more weight on firm value at time 2. Intuitively, this suggests the manager is *less* concerned with the short-term benefit from manipulating the firm's market value at time 1 and *more* concerned with long-term benefit from maximizing firm value at time 2. Furthermore, less lying at time 1 reduces the penalty assessed at time 2. This suggests increasing β_2 decreases the incentives for lying and distortion at time 1. Thus, the information partition should become finer as β_2 increases. Consistent with this intuition, Figure 18 shows that an increase in β_2 increases the equilibrium ratio r.

Figure 18. Sensitivity of r to β_2

Parameter values: $\beta_1 = 0.01$, $k = 0.5$, $\hat{k} = 1$

$$r(\beta_2) = \underset{r\in[0,1]}{\text{Solve}}\left\{.01 + .08r + .08r^2 + .01r^3 + \left(6.06\beta_2 + 6\beta_2^{\,2}\right)(r-1) = 0\right\}$$

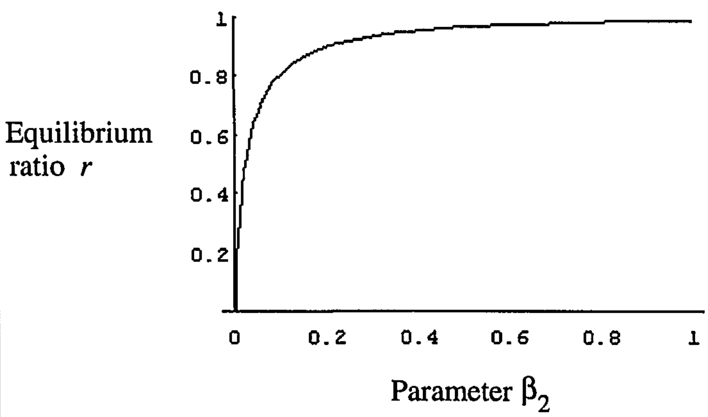


Figure 18 illustrates the sensitivity of r to β_2. The critical threshold for β_2 satisfies $\beta_1 - 6\,\beta_2(1+\beta_1+\beta_2)\hat{k} = 0$. Given the parameter β_1=0.01, k=0.5, and $\hat{k}$=1, assumed in this example, the threshold value for β_2 is 0.00165. For values of β_2 less than 0.00165, the value of r is not positive, which implies the incentives associated with time 2 are so weak that a pure pooling occurs. The ratio r increases as β_2 exceeds the threshold. When β_2 is 0.04 as in the previous examples, r is 0.63. As β_2 increases toward one, Figure 17 shows the equilibrium r continues to increase toward r=1. Values of β_2 greater than 1 are not presented, because they imply firm managers are paid more than 50 percent of the firm's available cash at time 2.

An increase in parameter value β_1 implies the firm manager's compensation function places more weight on firm value at time 1. Intuitively, this suggests the manager is *more* concerned with the short-term benefit from manipulating the firm's market value at time 1 and *less* concerned with long-term benefit from maximizing firm value at time 2. Thus, increasing β_1 should increase the incentives to lie and decrease the fineness of the information partition. If β_1 is sufficiently large, then every firm mimics the highest type in a pure pooling equilibrium. Consistent with this intuition, Figure 19 shows that an increase in β_1 decreases ratio *r*.

Figure 19. Sensitivity of *r* to β_1

Parameter values: $\beta_2 = 0.04,\ \ k = 0.5,\ \ \hat{k} = 1$

$$r(\beta_1) = \underset{r \in [0,1]}{\text{Solve}} \left\{0.2496(r-1) + \beta_1(.76 + 8.24r + 8r^2 + r^3) = 0\right\}$$

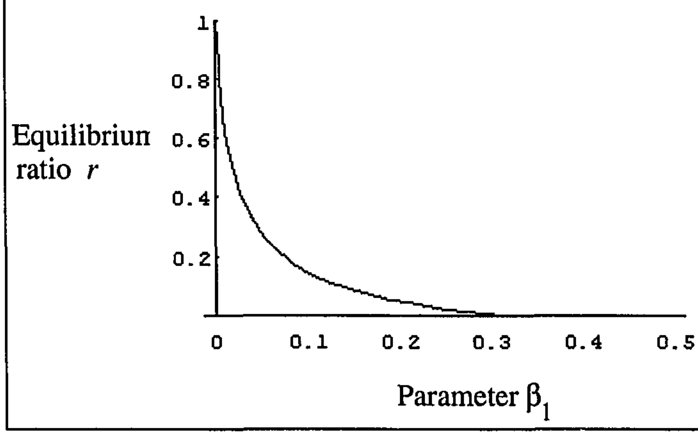


Figure 19 illustrates the sensitivity of *r* to β_1. When β_1 is zero, the firm manager has no incentive to influence investors' expectations at time 1, and the equilibrium *r* is one. When *r* is one, each interval is a single point and no mimicking occurs. As β_1 increases, the equilibrium ratio *r* decreases. When β_1=0.01 as in the prior examples, the value of *r* is 0.63. The critical threshold for β_1 is defined such that $\beta_1 - 6\beta_2(1+\beta_1+\beta_2)\hat{k} = 0$. When β_1 exceeds the critical threshold, then *r* is not positive. Given the parameter values assumed in Figure 19, the threshold value for β_1 is 0.328. For values of β_1 greater than 0.328, the incentives for lying at time 1 are so strong that a pure pooling occurs.

CHAPTER 5

SUMMARY AND DISCUSSION OF RESULTS

5.1. Model summary

This dissertation is motivated by the empirical evidence that when firms announce restructurings of operating assets, the stock market reaction may be positive, negative or indifferent. When firms have significant operations in high-technology, international marketing, or other rapidly changing product market environments, the restructuring announcements may be more useful indicators of future cash flow than audited reports of accounting earnings from past periods. External auditors are required to independently inspect evidence of past performance to verify reported accounting earnings. However, restructuring announcements are supported by management assertions about future performance in the next operating period that cannot be independently verified until that period is over. Therefore, the question arises under what conditions should investors interpret restructuring announcements as useful signals of future performance.

The stylized model constructed in Chapter 3 is based on two important SEC rules related to restructuring announcements. First, restructuring announcements are mandatory, since the SEC requires registered firms to discuss prospective resource allocation plans in their annual management discussion and analysis (MD&A). Second, SEC Rule 10b-5 allows investors to claim damages from firms that make false announcements. In my model the Rule 10b-5 penalties depend on the difference between the announcement $\hat{a}$ at the beginning of the operating period (time 1) and the actual allocation a at the end of the period (time 2).

I set up a simple model of a firm with two operating divisions, ESTABLISHED and DEVELOPMENT. At time 1 the firm manager privately observes the firm's type, θ, the future cash flow rate of return on resources invested in DEVELOPMENT. The manager makes two concurrent decisions at time 1, an allocation of a resources to DEVELOPMENT and an announcement $\hat{a}$. Lying is defined as the difference between a and $\hat{a}$. Investors observe the announcement $\hat{a}$, revise their beliefs about the firm's type, and price the firm at $V_1(\hat{a})$.

At time 2 the firm realizes cash flow $Z(\bullet)$, which is a function of the allocation a and productivity parameter θ. This cash is distributed as follows. First, a Rule 10b-5 penalty is paid as a deadweight loss if the firm lied at time 1. Second, incentive compensation $W(\bullet)$ is paid to the firm manager. Finally, residual value $V_2(\bullet)$ is paid to the investors.

The firm manager's compensation is a linear combination of the firm's stock price at time 1 and time 2, $V_1(\hat{a})$ and $V_2(\bullet)$, respectively. Lying at time 1 may benefit the manager by increasing investor expectations and stock price at time 1. Lying is costly at time 2 when the amount of lying is discovered and the penalty is paid. The manager's decision problem is to choose an announcement and allocation pair at time 1 that maximizes his incentive compensation.

Investors' respond by pricing the firm stock at its expected value considering the expected amount of lying and distortion. In equilibrium, the investors' beliefs are consistent with the firm's equilibrium strategy. If there are some announcements that no firm type would choose in equilibrium, then the investors' beliefs for those out-of-equilibrium announcements are sufficiently severe that no firm deviates to those announcements.

5.2. Discussion of results

The principal result of chapter 4's analysis is that the stock price reaction to restructuring announcements depends on the magnitude of the Rule 10b-5 penalty rates (parameter $\hat{k}$). When penalty rates are low, all firms mimic the restructuring announcement of the highest type, and there is no stock price reaction to the announcements. When penalty rates are sufficiently high, restructuring announcements are noisy signals of future cash flows, and investor reaction depends on the firm's announcement. The numerical examples of Chapter 4 are summarized in Table 1.

Table 1. Comparison of equilibria examples.

Parameter values for all examples: $\beta_1 = 0.01$, $\beta_2 = 0.04$, $k = 0.5$, $\tilde{\theta} \sim [0,1]$

Cash-maximizing allocation strategy is $a^*(\theta) = \theta$.

Average lie is $E_{\tilde{\theta}}[\hat{a}(\theta) - a(\theta)]$. Average distortion is $E_{\tilde{\theta}}[a(\theta) - a^*(\theta)]$.

Market return is defined as $R(\hat{a}) \equiv \dfrac{V_1(\hat{a}) - V_0}{V_0}$.

Equilibrium and firm's strategy	Parameters that differ among examples	Average lie	Average distortion	Reaction to $\hat{a} = 0$, $R(0)$	Reaction to $\hat{a} = 1$, $R(1)$
First-best $\{\hat{a}^*(\theta), a^*(\theta)\}$	Type is public Any $\hat{k} \geq 0$	0	0	-0.308	+0.615
Pure pooling $\{\hat{a}^{HP}(\theta), a^{HP}(\theta)\}$	$\hat{k} = 0.035$	0.468	0.033	-0.293 ($\hat{a} = 0$ is not observed in this pure pooling.)	0 ($\hat{a} = 1$ is the only $\hat{a}$ observed in this pure pooling.)
Partial pooling $\{\hat{a}^{rPP}(\theta), a^{rPP}(\theta)\}$	$\hat{k} = 1$ $r = 0.63$	0.038	0.076	-0.298	+0.304
Partial pooling $\{\hat{a}^{r\infty}(\theta), a^{r\infty}(\theta)\}$	$\hat{k} = \infty$ $r = 0.71$	0	0.084	-0.299	+0.364

The first-best case assumes the firm's type is publicly observed by both the firm and the investors. In this environment, the firm is unable to deceive the investors about its type, because the investors observe type at the same time as the firm. Proposition 1 shows that in the first-best case with k equal to one-half, the allocation $a^*(\theta) = \theta$ maximizes $Z(\bullet)$, the total available cash at time 2, and $W(\bullet)$, the firm manager's incentive compensation. Distortion is defined as an allocation that deviates from the allocation $a^*(\theta)$. In the first-best case there is no distortion and no lying.

When the firm privately observes its type and the penalty for lying is small, such as $\hat{k} = 0.035$, there is a pure pooling equilibrium in which all types mimic the $\hat{a} = 1$ announcement of the highest type. When the investors observe the $\hat{a} = 1$ announcement, there is no reaction because the investors expected all types to announce $\hat{a} = 1$. Any type that deviated to some out-of-equilibrium announcement, $\hat{a} < 1$, would have a negative return. To achieve zero lying, the firm would pick an allocation equal to the announcement, $a = \hat{a} = 1$. To achieve zero distortion, the firm would pick an allocation equal to its cash-maximizing allocation, $a^*(\theta)$. When the penalty for lying is small, such as $\hat{k} = 0.035$, the firm's best allocation has a large amount of lying and relatively small distortion.

The intuition of the pure pooling equilibrium is illustrated by an example where all firms can make the same ambiguous announcements such as, "The Company is restructuring operations to position itself as a leader in high-technology markets." Investors expect firms to make these announcements, and do not react when they are made. If the Rule 10b-5 penalty is low or not enforced, then every firm carries out an operating plan that may differ significantly what was announced.

When the penalty rate for lying is sufficiently high, such as $\hat{k} = 1$, there is a partial pooling equilibrium, where the firm's type and allocation are partially revealed to the investors. In the partial pooling equilibrium, the range of feasible types is partitioned into many intervals. Each interval is characterized by a unique announcement. In the partial pooling equilibrium, the investors observe the announcement, correctly infer the interval of types making that announcement, and correctly infer the average allocation those types make. High type firms (above average productive opportunities) make the highest announcements and receive a significant positive return. The lowest type firms make the lowest announcements, and receive negative returns.

In the partial pooling equilibrium the interval length decreases as the type decreases. The highest interval consists of high and moderately high type firms that choose to mimic the highest type firm and announce a "good news" restructuring. A long interval of types announcing good news is a relatively imprecise signal that the firm type is above average. The second highest interval is shorter than the highest interval and is essentially an "average news" announcement. Stepping progressively down to lower intervals, the announcements are progressively worse "bad news;" and interval lengths are progressively shorter. The incremental change in investor price response, $V_1(\hat{a})$, between the in-equilibrium announcements decreases as the announcements become smaller. The change in investor response is essentially zero between the lowest feasible announcement $\hat{a} = 0$ and an announcement slightly greater than zero. For the lowest type, $\theta = 0$, the very small benefit of mimicking a slightly higher type is less than the penalty for lying. Thus, the lowest type prefers to make the "worst news" announcement, $\hat{a} = 0$, and reveal itself as the lowest type. Since interval length is shorter as the announcements move down from good news to worst news, the announcements are least precise for good news and more precise as the news is worse.

The investors in my model are risk-neutral. In equilibrium, the investors' expected profits are zero. Risk-neutrality implies the investors in my model do not prefer more precise announcements over less precise announcements.

When the exogenous penalty rate for lying is increased, intuition suggests firms will lie less. Comparing the partial pooling with $\hat{k} = 1$ to the pure pooling with $\hat{k} = 0.035$ supports the intuition that when $\hat{k}$ increases, the average lie decreases. In the extreme case where $\hat{k}$ approaches infinity, lying is eliminated.

A change in the penalty for lying induces firms to increase distortion. The three examples in Table 1 with values of $\hat{k}$ equal to 0.035, 1, and infinity have average distortion of 0.033, 0.076, and 0.084, respectively. The firm's compensation-maximizing allocation is a tradeoff between its announcement, $\hat{a}$,and cash-maximizing allocation, $a^*(\theta)$. When the penalty for lying increases, the firm maximizes compensation by moving its allocation closer to the announcement and away from its cash-maximizing allocation. Thus, in this model a change in the penalty for false announcements induces firms to change their operating policy.

The partial pooling equilibrium examples provide some results consistent with the empirical observations of restructuring announcements in the real world. Firms with better prospects make announcements that result in higher returns. Firms with the worst prospects make announcements that result in the most negative returns. Some firms mimic the announcements of other firms making it more difficult for investors to precisely estimate future cash flows.

The sensitivity results of my analysis in section 4.5.5 suggest some hypotheses that could be empirically tested. If there are certain industry groups or time periods that have significantly higher reporting penalties (high $\hat{k}$), then my model predicts a stronger market reaction to restructuring announcements. Firms with executive compensation contracts emphasizing current share price (high β_1) are more likely to make false restructuring announcements that are not carried out. Testing these hypotheses depends on collecting a sufficiently large sample of restructuring announcements, quantifying the announcement magnitude, and controlling for confounding events such as changes in corporate control.

5.3. Policy implications

The relation between distortion and lying in my partial pooling equilibrium analysis has some policy implications. Increasing the effective penalty for Rule 10b-5 violations is represented in this model by an increase in the penalty rate $\hat{k}$. The results of the analysis in the partial pooling equilibrium in section 4.5.4 shows that as $\hat{k}$ increases, the amount of lying decreases and distortion increases. The policy implication of this result is that an increase in mandatory reporting requirements may induce firms to make operating decisions that do not maximize the firm's future cash flow.

This result occurs because the model assumes firms are sued for false announcements; but firm managers are not sued if they make an operating decision that does not maximize cash flow. In my model at time 1 the investors are uncertain about the productivity θ, but at time 2 they know a, $\hat{a}$, and $Z(\theta,\hat{a},a)$. At time 2 the investors could invert $Z(\theta,\hat{a},a)$ and determine θ and the cash-maximizing allocation $a^*(\theta) = \theta$ with certainty. Therefore, at time 2, the investors could infer the amount of distortion. However, at time 2 the model does not allow investors to directly assess a personal penalty on managers who distorted. Distortion does reduce the firm manager's compensation, because distortion is an opportunity cost to both the manager and the investors. With a distorted allocation the cash available for distribution, $Z(\theta, \hat{a}, a)$, is less than it would be with the cash-maximizing allocation $a^*(\theta) = \theta$. In equilibrium, the investors' average profits at time 2 are zero, because they adjust their pricing response at time 1 for the expected amount of distortion.

I recognize that in the real world investors and regulatory agencies can penalize managers for either false disclosures (lying) or operating decisions that do not maximize resource efficiency (distortion). I model only one penalty for lying: a Rule 10b-5 penalty that reduces firm value for both the manager and the investors. In the real world managers who do not manage resources efficiently can be sued personally or fired for failing to carry out their fiduciary duty to the stockholders. A possible extension of the model set-up is to add a personal penalty on managers who distort.

The interplay between lying and distortion in my equilibrium analysis is related to the debate on the proper role for financial disclosure regulation. Improving financial disclosure has been justified as means to help external investors make better decisions on how to allocate capital among different firms. With this justification the regulators have required firm managers to disclose more internal operating plans for the investors to use in estimating future cash flow. Investors can use Rule 10b-5 to sue firms and related parties that make inadequate disclosures. My analysis suggests that requiring disclosures to help investors allocate capital between firms may have an adverse impact on how those firms allocate operating resources within the firm. Perhaps regulators should be more concerned with motivating the accurate reporting of past performance so that stockholders could take action against managers with demonstrated poor performance.

When a firm manager has an incentive to maximize current share price as in my model, he may make announcements and operating decisions that mimic the strategy of the high type "industry leaders", rather than maximize the efficient allocation of resources given his own firm's unique characteristics. Suppose the SEC or FASB mandates firms make more specific disclosures about future resource allocations. If more specific prospective announcements are made by firms, then hostile plaintiff attorneys can more readily extract large settlements under Rule 10b-5 from firms that do not carry out their announcements. If litigation cost is potentially large relative to total firm value, the firm must adjust its operating policy to reduce litigation costs. Thus, with higher reporting penalties firm managers act defensively to increase current share price and reduce litigation costs, rather than focus on maximizing future cash flow of the firm from operations.

The Xerox case study discussed on the first page of this dissertation illustrates the phenomenon of firms announcing restructurings that *ex post* are not cash-maximizing. In the Xerox case the firm announced a restructuring in 1989 that had a short-term positive stock return, but did not result in a long-term increase in earnings or share price. Four years later, Xerox restructured again.

Lying and distortion could be eliminated from the model by changing some assumptions. If the firm manager and investors observed the same public information about future prospects (first-best environment), then lying and distortion will not benefit the manager. If the compensation contract were changed to eliminate the incentive on current share price (set $\beta_1 = 0$), then the firm would focus entirely on maximizing future cash flow and would have no incentive to lie. If firms were required to announce projections of future cash flow, denoted $\hat{Z}$, and incurred a reporting penalty based on the difference between projected and actual cash flow, denoted $\hat{Z} - Z(\bullet)$, then both lying and distortion would be eliminated for a sufficiently high reporting penalty.

5.4. Limitations

The assumptions of this model may limit its ability to explain stock market reaction to restructuring announcements observed in the real world.

This model assumes the managers and investors agree on the direction of "good news." I assumed that a credible announcement of moving resources toward DEVELOPMENT (higher values of $\hat{a}$) is interpreted as better than moving resources toward ESTABLISHED (lower values of $\hat{a}$). Without this assumption, managers cannot anticipate the direction of investor reaction to announcements. In the real world, there may be cases where the firm managers and investors disagree over what is good news. For example, a steel firm may believe consolidating operations at its biggest steel plant is good news. Investors may disagree with the firm and interpret the steel plant expansion announcement as an unfavorable sign the firm is concentrating resources in a slow-growth manufacturing industry.

My model assumes shareholders of the firm have conflicting preferences over the time they will liquidate their stock. To satisfy shareholders who want to liquidate at two different points of time, the manager's incentive compensation contract in this model is a linear combination of the firm's current and future stock price. This contract motivates the manager to strategically choose its announcement to influence investors' expectations and current stock price. Shareholders who wish to liquidate at time 1 want to sell their stock at the highest possible price and prefer that managers not reveal bad news. If the shareholders wanted to motivate to the manager to maximize firm value at time 2, then they would change the contract so that the manager's compensation depended on the firm's operating cash flows observable at time 2. Thus, the model may not be useful in predicting returns if a firm's incentive compensation contract does not depend on current stock price.

Quadratic production and penalty functions were assumed in this model for the purpose of obtaining closed form analytical solutions. To determine whether the results are dependent on the functional forms assumed, the analysis could be extended to other production and penalty functions.

APPENDIX. Proofs.

Lemma 1. Given assumptions (A-3), (A-4), and (A-5),

$$V_1(\hat{a}) = \frac{E_{\tilde{\theta}}\left[Z(\bullet)|\hat{a}\right]}{1+\beta_1+\beta_2} \tag{1}$$

$$\text{and } W(\bullet) = \frac{\beta_1}{1+\beta_2}\frac{E_{\tilde{\theta}}\left[Z(\bullet)|\hat{a}\right]}{1+\beta_1+\beta_2} + \frac{\beta_2}{1+\beta_2}Z(\bullet) \tag{2}$$

Discussed in text at page 50.

Proof: Begin with assumption (A-5).

$$V_1(\hat{a}) \equiv E_{\tilde{\theta}}\left[V_2(\bullet)|\hat{a}\right]$$

Observe $E_{\tilde{\theta}}\left[V_1(\hat{a})|\hat{a}\right] = E_{\tilde{\theta}}\left[E_{\tilde{\theta}}\left[V_2(\hat{a})|\hat{a}\right]|\hat{a}\right] = E_{\tilde{\theta}}\left[V_2(\hat{a})|\hat{a}\right] = V_1(\hat{a})$

Substitute assumptions (A-3) and (A-4) into (A-5).

$$\begin{aligned} V_1(\hat{a}) &= E_{\tilde{\theta}}\left[V_2(\hat{a})|\hat{a}\right] = E_{\tilde{\theta}}\left[Z(\bullet) - \beta_1 V_1(\hat{a}) - \beta_2 V_2(\bullet)|\hat{a}\right] \\ &= E_{\tilde{\theta}}\left[Z(\bullet)|\hat{a}\right] - \beta_1 E_{\tilde{\theta}}\left[V_1(\hat{a})|\hat{a}\right] - \beta_2 E_{\tilde{\theta}}\left[V_2(\bullet)|\hat{a}\right] \\ &= E_{\tilde{\theta}}\left[Z(\bullet)|\hat{a}\right] - \beta_1 V_1(\hat{a}) - \beta_2 V_1(\hat{a}) \end{aligned}$$

Collect terms of $V_1(\hat{a})$ and divide by $(1 + \beta_1 + \beta_2)$ to yield result (1).

Substitute (A-3) and result (1) into (A-4).

$$W(\bullet) = \beta_1 V_1(\hat{a}) + \beta_2 Z(\bullet) - \beta_2 W(\bullet)$$

$$(1+\beta_2)W(\bullet) = \beta_1 \frac{E_{\tilde{\theta}}\left[Z(\bullet)|\hat{a}\right]}{1+\beta_1+\beta_2} + \beta_2 Z(\bullet)$$

Divide both sides by $(1 + \beta_2)$ to yield result (2). ■

Proposition 1. Firm cash at time 2, denoted $Z(\theta, \hat{a}, a)$, is maximized when the firm chooses allocation $a^*(\theta) = \frac{1}{2} - \frac{1}{4k} + \frac{\theta}{2k}$ and announcement $\hat{a}^*(\theta) = a^*(\theta)$.

Discussed in text at page 62.

Proof: The optimization problem is $\underset{\hat{a},a}{\text{Max}}\, Z(\theta, \hat{a}, a)$.

Assumption (A-2) from the set-up section defines $Z(\bullet)$.

$$Z(\theta, \hat{a}, a) \equiv \theta a + \frac{1}{2}(1-a) - k\left(a - \frac{1}{2}\right)^2 - \hat{k}(\hat{a} - a)^2$$

First-order conditions are found by differentiating with respect to a and $\hat{a}$.

$$\frac{\partial Z(\bullet)}{\partial a} = \theta - \frac{1}{2} - 2k\left(a - \frac{1}{2}\right) + 2\hat{k}(\hat{a} - a)$$

Solving $\frac{\partial Z(\bullet)}{\partial a} = 0$, gives $a^* = \dfrac{\theta - \frac{1}{2} + k + 2\hat{k}\hat{a}}{2\left(k + \hat{k}\right)}$

$\frac{\partial Z(\bullet)}{\partial \hat{a}} = -2\hat{k}(\hat{a} - a)$. Solving $\frac{\partial Z(\bullet)}{\partial \hat{a}} = 0$ gives $\hat{a}^* = a^*$.

Combining first-order conditions gives $a^* = \dfrac{\theta - \frac{1}{2} + k + 2\hat{k}a^*}{2\left(k + \hat{k}\right)}$.

Solving for a^* gives $a^*(\theta) = \frac{1}{2} - \frac{1}{4k} + \frac{\theta}{2k}$, and $\hat{a}^*(\theta) = a^*(\theta)$.

The solution to the first-order conditions is a maximum when the Hessian matrix is negative definite: $Z_{\hat{a}\hat{a}} < 0$, and $|H| > 0$.

The second derivatives for $Z(\bullet)$ are $Z_{\hat{a}\hat{a}} = -2\hat{k} < 0$; $Z_{aa} = -2(k+\hat{k}) < 0$; and $Z_{\hat{a}a} = Z_{a\hat{a}} = 2\hat{k} > 0$.

$$|H| = \det\begin{vmatrix} Z_{\hat{a}\hat{a}} & Z_{\hat{a}a} \\ Z_{a\hat{a}} & Z_{aa} \end{vmatrix} = Z_{\hat{a}\hat{a}} Z_{aa} - Z_{a\hat{a}} Z_{\hat{a}a} = 4k\hat{k} + 4\hat{k}\hat{k} - 4\hat{k}\hat{k} = 4k\hat{k} > 0$$

Thus, the solution from the first-order condition is the maximum.

The optimal allocation function $a^*(\theta)$ must be restricted so that the allocation is in the feasible allocation interval [0, 1].

The upper boundary $a^*(\theta) \leq 1$ is binding when $\theta \geq \frac{1}{2} + k$.

The lower boundary $a^*(\theta) \geq 0$ is binding when $\theta \leq \frac{1}{2} - k$.

Note that if $k=\frac{1}{2}$, then $a^*(\theta) = \theta$ and $0 \leq a^*(\theta) \leq 1$ for all $\theta \in [0,1]$. ■

Proposition 2. If the firm's type and allocation are publicly observable, the firm's equilibrium strategy is $\hat{a}(\theta) = a(\theta) = a^*(\theta)$ for all $\theta \in [0, 1]$, which is defined as no lying and no distortion.

Discussed in text at page 66.

Proof: When the firm's type θ, announcement $\hat{a}$, and allocation a are publicly observable, then the investors know $Z(\bullet) = Z(\theta, \hat{a}, a)$. Substituting into result (1) from Lemma 1 gives the investors' response as a function of all observable information: $V_1(\theta, \hat{a}, a) = \dfrac{Z(\theta, \hat{a}, a)}{1 + \beta_1 + \beta_2}$.

Substituting into result (2) from Lemma 1 gives the firm's compensation function:

$$W(\theta, a, \hat{a}, V_1(\bullet)) = \frac{\beta_1}{1+\beta_2} \frac{Z(\theta, \hat{a}, a)}{1+\beta_1 + \beta_2} + \frac{\beta_2}{1+\beta_2} Z(\theta, \hat{a}, a)$$

$$= \frac{\beta_1 + \beta_2}{1+\beta_1 + \beta_2} Z(\theta, \hat{a}, a)$$

Thus, the firm's compensation maximization problem is

$$\underset{\hat{a}, a}{\text{Max}}\, W(\theta, \hat{a}, a, V_1(\hat{a})) = \frac{\beta_1 + \beta_2}{1+\beta_1 + \beta_2} \underset{\hat{a}, a}{\text{Max}}\, Z(\theta, \hat{a}, a)$$

This problem is equivalent to the optimization problem solved in Proposition 1. The solution is $\hat{a}(\theta) = a(\theta) = a^*(\theta)$ for all $\theta \in [0, 1]$. This result is described as *no distortion* and *no lying.* ■

Corollary 2.1. A full-revelation equilibrium exists if and only if the firm's equilibrium strategy is $\hat{a}(\theta) = a(\theta) = a^*(\theta)$ for all $\theta \in [0, 1]$.

Discussed in text at page 71.

Proof of sufficiency: Suppose the outcome is

$\hat{a}(\theta) = a(\theta) = a^*(\theta) = \frac{1}{2} - \frac{1}{4k} + \frac{\theta}{2k}$ for all $\theta \in [0, 1]$. Since k=0.5 by (A-7), the outcome is $\hat{a}^*(\theta) = a^*(\theta) = \theta$ and the inverse function, $\Phi(\hat{a}) = \hat{a}$. Thus, $\hat{a}^*(\theta)$ fully reveals the type.

Proof of necessity: Let $\{\hat{a}_o, a_o\}$ represent the strategy of firm type θ_o and suppose the announcement is fully-revealing and has a nonzero amount of lying. This implies $\Phi(\hat{a}_o) = \theta_o$. Define the amount of lying as $\delta \equiv \hat{a}_o - a_o \neq 0$.

If this is an equilibrium, then the investors' expectation of the allocation is consistent with the firm's actual allocation.

$$E[a|\hat{a}_o] = a\left(E[\tilde{\theta}|\hat{a}_o]\right) = a(\theta_o) = \hat{a}_o - \delta = a_o$$

This implies the investors correctly infer future cash flow with no uncertainty.

$$\begin{aligned} E_{\tilde{\theta}}[Z(\bullet)|\hat{a}_o] &= Z(\theta_o, \hat{a}_o, a_o) \\ &= \theta_o a_o + \frac{1}{2}(1-a_o) - k\left(a_o - \frac{1}{2}\right)^2 - \hat{k}(\hat{a}_o - a_o)^2 \\ &= Z(\theta_o, a_o, a_o) - \hat{k}\delta^2 \end{aligned}$$

Substituting into result (2) from Lemma 1 and simplifying yields the following.

$$W(\theta_o, \hat{a}_o, a_o, V_1(\hat{a}_o)) = \frac{\beta_1}{1+\beta_2}\frac{Z(\theta_o, \hat{a}_o, a_o)}{1+\beta_1+\beta_2} + \frac{\beta_2}{1+\beta_2}Z(\theta_o, \hat{a}_o, a_o)$$

$$= \frac{\beta_1 + \beta_2}{1 + \beta_1 + \beta_2} \left\{ Z(\theta_o, a_o, a_o) - \hat{k}\delta^2 \right\}$$

Firm compensation is maximized by choosing $\delta = 0$, which implies $\hat{a}_o = a_0$.

Thus, the firm will not lie and its allocation problem simplifies to the following.

$$\underset{a}{\text{Max}}\, W(\theta, a, a, V_1(a)) = \frac{\beta_1 + \beta_2}{1 + \beta_1 + \beta_2} \underset{a}{\text{Max}}\, Z(\theta, a, a)$$

From Proposition 1 the solution is $\hat{a}(\theta) = a(\theta) = a^*(\theta)$ for all $\theta \in [0, 1]$. ■

Proposition 3. If the firm privately observes its type, then a fully revealing equilibrium does not exist.

Discussed in text at page 72.

Proof by contradiction:

Suppose a fully-revealing equilibrium exists. Corollary 2.1 and assumption (A-7) imply investors believe the firm's strategy is $\hat{a}^*(\theta) = a^*(\theta) = \theta$ for all $\theta \in [0, 1]$. The investors use the inverse function $\Phi(\hat{a}) = \hat{a}$ to infer the firm's type. The investors' response is

$$V_1^*(\hat{a}) = \frac{Z(\Phi(\hat{a}), \hat{a}, \hat{a})}{1 + \beta_1 + \beta_2} = \frac{0.375 + 0.5\hat{a}^2}{1 + \beta_1 + \beta_2}.$$

Given this investors' response and the firm's allocation $a^*(\theta)$, does any type choose to change its announcement? The highest type $\theta=1$ has the strategy $\hat{a}^*(1) = a^*(1) = 1$. Allow firms to choose the full-revelation announcement $\hat{a}^*(\theta)$ or the mimicking announcement $\hat{a} = 1$.

Define the benefit of mimicking as the increased payoff from the change in investors' response at time 1. $B(\theta) \equiv \frac{\beta_1}{1 + \beta_2}\{V_1^*(\hat{a}^*(1)) - V_1^*(\hat{a}^*(\theta))\}$.

Substituting and simplifying, $B(\theta) = \frac{\beta_1(1-\theta)(1+\theta)}{2(1 + \beta_2)(1 + \beta_1 + \beta_2)}$.

Define the loss from mimicking as the decrease in payoff related to the penalty at time 2. $L(\theta) \equiv \frac{\beta_2}{1 + \beta_2}\{Z(\theta, \hat{a}^*(\theta), a^*(\theta)) - Z(\theta, \hat{a}^*(1), a^*(\theta))\}$

Substituting and simplifying, $L(\theta) = \frac{\beta_2}{1+\beta_2}\hat{k}\{\hat{a}*(1) - \hat{a}*(\theta)\}^2 = \frac{\beta_2 \hat{k}(1-\theta)^2}{(1+\beta_2)}$.

Find the firm types that are indifferent, by solving $B(\theta) = L(\theta)$ for θ.

The solutions are θ=1 and $\theta = 1 - \varepsilon_o$; where $\varepsilon_o \equiv \frac{2\beta_1}{\beta_1 + 2\beta_2 \hat{k}(1+\beta_1+\beta_2)}$.

Define the open interval $M \equiv \{\theta : 1 - \varepsilon_o < \theta < 1\}$.

To show that the benefits of deviating exceed the loss for types in M, consider the interval's midpoint, $\theta_d = 1 - \frac{1}{2}\varepsilon_o = 1 - \frac{\beta_1}{\beta_1 + 2\beta_2(1+\beta_1+\beta_2)}$.

$$B(\theta_d) - L(\theta_d) = \frac{\beta_1^2}{2(1+\beta_2)(1+\beta_1+\beta_2)\{\beta_1 + 2\beta_2 \hat{k}(1+\beta_1+\beta_2)\}} > 0$$

Therefore, type θ_d strictly prefers to deviate. Since interval M is bounded by the only two types which are indifferent between deviating and full revelation and at least one type strictly prefers to deviate, then all types in M strictly prefer to deviate.

Thus, the full revelation equilibrium unravels. ■

Corollary 3.1. If the firm privately observes its type and $\hat{k}$ is exogenously increased to an arbitrarily large positive finite value, there always exists some interval of types that lie.

Discussed in text at page 75.

Proof: The interval *M* defined in Proposition 3 contains a continuous interval in $[0,1]$ whenever $1-\varepsilon_0<1$. Observe $\varepsilon_0 = \dfrac{2\beta_1}{\beta_1 + 2\beta_2\hat{k}(1+\beta_1+\beta_2)} > 0$ for any positive finite $\hat{k}$. However, as $\hat{k}$ approaches infinity, ε_0 approaches zero, and *M* degenerates from a continuous interval to the single point $\{\theta = 1\}$. ■

Corollary 3.2. If the firm privately observes its type and $\hat{k}$ approaches positive infinity, then the first-best outcome is not an equilibrium.

Discussed in text at page 77.

Proof by contradiction:

Step 1. Suppose the investors believe the firm's strategy is first-best, then determine their pricing response.

If the investors believe the firm's strategy is first-best (no lying and no distortion), then identify the investors' response.

Given the parameter restriction *k*=0.5, the firm's first-best allocation function, $\hat{a}^*(\theta)$, can be inverted to reveal a unique type.

$$\mu(\tilde{\theta}|\hat{a}) = \Phi(\hat{a}) = \hat{a}$$

Investors believe the firm's announcement is truthful, $\hat{a} = a$.

Let $V_1^*(\hat{a})$ denote the investors' valuation at time 1 given their belief that the firm's strategy is first-best. $V_1^*(\hat{a}) = \dfrac{Z(\Phi(\hat{a}), \hat{a}, \hat{a})}{1 + \beta_1 + \beta_2} = \dfrac{0.375 + 0.5\,\hat{a}^2}{1 + \beta_1 + \beta_2}$

Step 2. Given the investors' valuation $V_1^*(\hat{a})$, identify the firm's best-response.

Let $\{\hat{a}^{br}(\theta), a^{br}(\theta)\}$ denote the firm's best-response allocation given $V_1^*(\hat{a})$.

The firm's allocation problem is $\underset{\{\hat{a},a\}}{\text{Max}}\ W(\theta, \hat{a}, a, V_1^*(\hat{a}))$ subject to $\hat{k} = +\infty$.

Assume the firm's announcement is truthful, $\hat{a} = a$.

Thus, the firm's problem becomes $\underset{\hat{a}\in[0,1]}{\text{Max}}\ W(\theta, \hat{a}, \hat{a}, V_1^*(\hat{a}))$.

Solving the first-order condition yields the firm's best-response strategy as

$$\hat{a}^{br}(\theta) = \text{Min}\left\{1,\ \frac{\beta_2(1+\beta_1+\beta_2)}{\beta_2(1+\beta_1+\beta_2)-\beta_1}\theta\right\}$$

$$a^{br}(\theta) = \hat{a}^{br}(\theta)$$

Recall that distortion is defined as the difference between the firm's chosen allocation a and the first-best allocation is $a^*(\theta) = \theta$. For all types $1>\theta>0$ and $\beta_1 > 0$, the firm's best-response has a positive amount of distortion,

$$\hat{a}^{br}(\theta) - a^*(\theta) = \frac{\beta_1\theta}{\beta_2(1+\beta_1+\beta_2)-\beta_1} > 0 \text{ or } \hat{a}^{br}(\theta) - a^*(\theta) = 1-\theta > 0.$$

Step 3. Conclude that the first-best outcome is not an equilibrium. In Step 1 the investors believed there was no distortion and used the price response $V_1{}^*(\hat{a})$. However, step 2 shows that given $V_1{}^*(\hat{a})$ the best-response of nearly all firms does involve distortion. Thus, the investors' belief about the firm's strategy is not confirmed. ■

Investors' beliefs in the pure pooling equilibrium in Proposition 4

The pure pooling equilibrium in Proposition 4 is supported by the following investors' belief function. The belief function is specified for the unique announcement observed in equilibrium, $\hat{a} = 1$, and any out-of-equilibrium announcement, $\hat{a} < 1$. Announcements $\hat{a} > 1$ or $\hat{a} < 0$ are not allowed.

$$\mu\left(\tilde{\theta}\middle|\hat{a}\right) = \tilde{\theta} \sim U(0, \hat{a})$$

Suppose the investors observe the in-equilibrium announcement, $\hat{a} = 1$. Given the above belief function, the investors believe the type $\tilde{\theta}$ is uniformly distributed on the interval $[0,1]$, and has an expected value of 0.5. In the pure pooling equilibrium, investors believe all firm types will announce $\hat{a} = 1$. Thus, both the prior and posterior beliefs are uniform on $[0,1]$ when the in-equilibrium announcement, $\hat{a} = 1$, is observed.

Suppose the investors observe an out-of-equilibrium announcement, $\hat{a}$=0.5. The investors know that if firms were using the full revelation announcement strategy, $\hat{a}^*(\theta) = \theta$, then they could invert that announcement, $\Phi(0.5) = 0.5$, and infer the firm type $\theta = 0.5$. However, in this case the investors believe the out-of-equilibrium announcement means the firm's type could be any type less than or equal to 0.5. Thus, if $\hat{a}$=0.5 were observed, investors would believe θ is uniform on $[0, 0.5]$, and the expected type is 0.25, the mean of the interval $[0, 0.5]$.

Suppose the investors observe the announcement, $\hat{a}^*(0)$, the announcement the lowest type would make in the full revelation environment. After this announcement the investors believe the firm's type must be the lowest type, θ=0.

This investor belief function satisfies (A-6) at the end of Chapter 3 that requires the investors' posterior expectation of type to be an increasing function of the announcement. $E[\tilde{\theta}|\hat{a}=0]=0<E[\tilde{\theta}|\hat{a}=0.5]=0.25<E[\tilde{\theta}|\hat{a}=1]<0.5$

Table 2 summarizes the investors' beliefs and responses in formal notation.

Table 2. Investors' beliefs and response in the pure pooling of Proposition 4.
Equilibrium announcement set: $\hat{A}^e=\{\hat{a}=1\}$
Out-of-equilibrium announcement set: $\hat{A}^o=\{\hat{a}|\,\hat{a}\in[0,1]\text{ and }\hat{a}<1\}$
Announcement inverse function: $\Phi(\hat{a})=\hat{a}$

Observed announce-ment, $\hat{a}$	Investors' belief about types, $\mu(\tilde{\theta}|\hat{a})$	Investors' expecta-tion of type, $E[\tilde{\theta}|\mu(\tilde{\theta}|\hat{a})]$	Investors' belief about allocation, $a^{HP}(\theta)$	Investors' valuation response at time 1, $V_1^{HP}(\hat{a})$
$\hat{a}=1$	$U(0,1)$	$\frac{1}{2}$	$a^{HP}(\theta)=\frac{\theta+2\hat{k}}{1+2\hat{k}}$	$\frac{\int_0^1 Z(\theta,1,a^{HP}(\theta))d\theta}{1+\beta_1+\beta_2}$
$0<\hat{a}<1$	$U(0,\hat{a})$	$\frac{\hat{a}}{2}$	$a^{HP}(\theta)=\frac{\theta+2\hat{k}}{1+2\hat{k}}$	$\frac{\int_0^{\hat{a}} Z(\theta,\hat{a},a^{HP}(\theta))\frac{1}{\hat{a}}d\theta}{1+\beta_1+\beta_2}$
$\hat{a}=0$	$\{\theta=0\}$ (single point)	0	$\frac{2\hat{k}}{1+2\hat{k}}$	$\frac{Z\left(0,0,\frac{2\hat{k}}{1+2\hat{k}}\right)}{1+\beta_1+\beta_2}$

Proposition 4. If the firm privately observes its type and the penalty factor $\hat{k}$ is sufficiently small, such that $0 < \hat{k} < \frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$, there exists a pure pooling equilibrium in which all types mimic the highest type.

Discussed in text at page 78.

Proof:

Parameter values assumed:

$$\beta_1 > 0;\ \beta_2 > 0;\ k = \frac{1}{2};\text{ and } 0 < \hat{k} < \frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}.$$

Define the full revelation strategies in the same manner as Proposition 1:

$\hat{a}^*(\theta) = a^*(\theta) = \theta$ when k=0.5.

Investors' beliefs about type: $\mu(\tilde{\theta}|\hat{a}) = \tilde{\theta} \sim U(0,\hat{a})$

See interpretation of investors' beliefs on previous pages.

Investors' beliefs about firm's allocation: $a^{HP}(\theta) = \frac{\theta + 2\hat{k}}{1 + 2\hat{k}}$

Step 1 proves that the firm's best allocation conditional on its announcement is $a^{HP}(\theta)$. The investors observe $\hat{a}$, and posterior beliefs about the firm's type specified by $\mu(\tilde{\theta}|\hat{a})$.

Proposed equilibrium:

Firms mimic the highest type and announce $\hat{a}^{HP} = \hat{a}^*(1) = 1$ for all $\theta \in [0,1]$.

Firm's allocation is $a^{HP}(\theta) = \dfrac{\theta + 2\hat{k}}{1 + 2\hat{k}}$.

Investors' response is $V_1^{HP}(\hat{a}) = \dfrac{1}{1+\beta_1+\beta_2}\int_0^1 Z(\tilde{\theta}, \hat{a}, a^{HP}(\tilde{\theta}))\, d\mu(\tilde{\theta}|\hat{a})$

Evaluating $V_1^{HP}(\hat{a})$ using the functions $a^{HP}(\theta)$ and $\mu(\tilde{\theta}|\hat{a})$, the investors' response is

$$V_1^{HP}(\hat{a}) = \frac{\frac{3}{8} + \frac{\hat{a}^2}{6+12\hat{k}}}{1+\beta_1+\beta_2} \quad \text{for } \hat{a} \in [0,1]$$

Outline of Proof: This proof shows that given the assumed parameter values and beliefs, the firm's payoff from the proposed equilibrium announcement and allocation pair, $\{\hat{a} = 1, a^{HP}(\theta)\}$, dominates the payoff from any other feasible pair, $\{\hat{a}, a\}$. Since there are an infinite number of feasible allocations, the proof considers a pair with an out-of-equilibrium announcement, $\hat{a}_o$, and the best allocation conditional on that announcement, $a^c(\theta, \hat{a}_o)$. Step 1 finds $a^c(\theta, \hat{a}_o)$ as a function of the parameters.

Step 2 considers a deviation from the proposed equilibrium to $\{\hat{a}_o, a^c(\theta, \hat{a}_o)\}$ where $\hat{a}_o < 1$. Step 3 shows the investors' response to the firm's in-equilibrium strategy pair, $\{\hat{a}^*(1), a^{HP}(\theta)\}$, is the expected value of the firm.

Step 1. Show that if a firm announces $\hat{a}$, then its compensation-maximizing allocation is $a^c(\theta, \hat{a}) = \dfrac{\theta + 2\hat{k}\hat{a}}{1 + 2\hat{k}}$.

After announcing $\hat{a}$ and the investors' response, result (2) from Lemma 1 and assumption (A-7) gives the firm's compensation as

$$W\left(\theta, \hat{a}, a, V_1^{HP}(\hat{a})\right) = \frac{\beta_1}{1+\beta_2} V_1^{HP} + \frac{\beta_2}{1+\beta_2} Z(\theta, \hat{a}, a)$$

where $Z(\theta, \hat{a}, a) = \theta a + \frac{1}{2}(1-a) - \frac{1}{2}\left(a - \frac{1}{2}\right)^2 - \hat{k}(\hat{a} - a)^2$

The firm's allocation problem is $\underset{a}{\text{Max}}\ W\left(\theta, a, \hat{a}, V_1^{HP}(\hat{a})\right)$.

The first-order condition is solved as follows.

$$\frac{\partial W\left(\theta, \hat{a}, a, V_1^{HP}(\hat{a})\right)}{\partial a} = \theta - \frac{1}{2} - \left(a - \frac{1}{2}\right) - 2\hat{k}(a - \hat{a}) = 0$$

$$a = \frac{\theta + 2\hat{k}\hat{a}}{1 + 2\hat{k}}$$

$$\frac{\partial^2 W\left(\theta, \hat{a}, a, V_1^{HP}(\hat{a})\right)}{\partial a^2} = -\left(1 + 2\hat{k}\right) < 0$$

Since the second-order sufficient condition is satisfied, the first-order condition yields the choice of *a* that will maximize the firm's compensation.

Step 1 implies that the firm's payoff from the strategy pair $\{\hat{a}_o, a^c(\theta, \hat{a}_o)\}$ dominates the payoff from the pair $\{\hat{a}_o, a\}$ where $a \neq a^c(\theta, \hat{a}_o)$.

Step 2. Show no firm deviates from the proposed equilibrium, $\{\hat{a}=1, a^{HP}(\theta)\}$, to $\{\hat{a}_o, a^c(\theta,\hat{a}_o)\}$ where $0 \le \hat{a}_o < 1$.

Define $\Delta(\theta, \hat{a}_o)$ as the difference in firm's payoff between the equilibrium strategy $\{\hat{a}=1, a^{HP}(\theta)\}$ and the out-of-equilibrium $\{\hat{a}_o, a^c(\theta,\hat{a}_o)\}$.

$$\Delta(\theta,\hat{a}_o) \equiv W(\theta, 1, a^{HP}(\theta), V_1^{HP}(1)) - W(\theta, \hat{a}_o, a^c(\theta,\hat{a}_o), V_1^{HP}(\hat{a}_o))$$

If $\Delta(\theta, \hat{a}_o) > 0$, then the type θ firm strictly prefers the in-equilibrium pair $\{\hat{a}=1, a^{HP}(\theta)\}$ over the out-of-equilibrium pair, $\{\hat{a}_o, a^c(\theta, \hat{a}_o)\}$.

Substituting in results (1) and (2) from Lemma 1 yields

$$\Delta(\theta,\hat{a}_o) = \frac{\beta_1}{1+\beta_2}\{V_1^{HP}(\hat{a}*(1)) - V_1^{HP}(\hat{a}_o)\}$$

$$+ \frac{\beta_2}{1+\beta_2}\{Z(\theta,1,a^{HP}(\theta)) - Z(\theta,\hat{a}_o,a^c(\theta,\hat{a}_o))\}$$

$$\Delta(\theta,\hat{a}_o) = \frac{\beta_1}{(1+\beta_2)}\frac{(1-\hat{a}_o)(1+\hat{a}_o)}{6(1+\beta_1+\beta_2)(1+2\hat{k})} + \frac{\beta_2}{(1+\beta_2)}\frac{\hat{k}(\hat{a}_o-1)(\hat{a}_o+1-2\theta)}{(1+2\hat{k})}$$

Differentiating with respect to θ shows that $\Delta(\theta, \hat{a}_o)$ increases as θ increases.

$$\frac{\partial\Delta(\theta,\hat{a}_o)}{\partial\theta} = \frac{\beta_2}{(1+\beta_2)}\frac{\hat{k}}{(1+2\hat{k})}(1-\hat{a}_o) > 0 \quad \text{because } 1 > \hat{a}_o$$

Thus, $\Delta(\theta, \hat{a}_o) > \Delta(0, \hat{a}_o)$ for all $\theta > 0$.

This implies that the type who least strongly prefers the equilibrium is the lowest type, $\theta = 0$. If the lowest type does not deviate, then the other types will not deviate.

Evaluating and simplifying, $\Delta(0,\hat{a}_o)=\dfrac{(1-\hat{a}_o)(1+\hat{a}_o)\{\beta_1-6\beta_2\hat{k}(1+\beta_1+\beta_2)\}}{6(1+\beta_2)(1+\beta_1+\beta_2)(1+2\hat{k})}$

$\Delta(0,\hat{a}_o)>0$ because $1>\hat{a}_o$ implies $(1-\hat{a}_o)>0$ and the parameter restriction

$0<\hat{k}<\dfrac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$ implies $\{\beta_1-6\beta_2\hat{k}(1+\beta_1+\beta_2)\}>0$.

Thus, $\Delta(0,\hat{a}_o)>0$ for all $\hat{a}_o$ such that $0\le\hat{a}_o<1$.

Combining results, $\Delta(\theta,\hat{a}_o)>\Delta(0,\hat{a}_o)>0$ for all $\theta>0$.

The result $\Delta(\theta,\hat{a}_o)>0$ implies the payoff the type θ firm strictly prefers the in-equilibrium pair $\{\hat{a}=1, a^{HP}(\theta)\}$ over the out-of-equilibrium pair, $\{\hat{a}_o, a^c(\theta,\hat{a}_o)\}$.

Combining Step 1 and Step 2 shows that the type θ firm strictly prefers $\{\hat{a}=1, a^{HP}(\theta)\}$ over $\{\hat{a}_o, a\}$ where $\hat{a}_o\in[0,\hat{a}*(1))$ and $a\in[0,1]$.

Step 3. Given the firm's announcement and allocation strategy pair, $\{\hat{a}=1, a^{HP}(\theta)\}$, show the investors' response $V_1^{HP}(1)$ is a best-response.

$$V_1^{HP}(1) = \frac{E_{\hat{\theta}}\left[Z(\bullet)\middle|\hat{a}=1,\mu(\tilde{\theta}|\hat{a}=1)\right]}{1+\beta_1+\beta_2} \quad \text{from result (1) in Lemma 1}$$

$$= \frac{1}{1+\beta_1+\beta_2}\int_0^1 Z(\theta,1,a^{HP}(\theta))\,d\theta \quad \text{because } d\mu(\tilde{\theta}|\hat{a}*(1))=1$$

$$= \frac{1}{1+\beta_1+\beta_2}\left\{\frac{3}{8}+\frac{1}{6+12\hat{k}}\right\} \quad \text{after simplifying}$$

$$= V_1^{HP}(\hat{a}=1)$$

■

Investors' beliefs in the partial pooling equilibrium in Proposition 5

The partial pooling equilibrium in Proposition 5 is supported by the following investors' belief function. In this partial pooling the equilibrium announcement set consists of a countable sequence of discrete announcements, $\hat{A}^e = \{\ldots, r^{j+1}, r^j, r^{j-1}, \ldots, r^2, r, 1\}$. There are an uncountable number of feasible announcements in the out-of-equilibrium set, $\hat{A}^o = \{\hat{a} | \hat{a} \in [0,1] \text{ and } \hat{a} \notin \hat{A}^e\}$. The investors' belief function is specified for all feasible announcements, whether they are in or out of the equilibrium set.

The equilibrium announcement set defines a partition of the feasible types.

$$\{\ldots, (r^{j+2}, r^{j+1}], (r^{j+1}, r^j], (r^j, r^{j-1}], \ldots, (r^2, r], (r, 1]\}$$

The jth interval in the partition is $(r^{j+1}, r^j]$, and is characterized by the equilibrium announcement r^j. The investors' belief function maps a feasible announcement $\hat{a}$ into one of these intervals.

$$\mu(\tilde{\theta}|\hat{a}) = \begin{cases} \tilde{\theta} \sim U(r,1) & \text{if } \hat{a} = 1 \\ \tilde{\theta} \sim U(r^2, r) & \text{if } \hat{a} \in [r,1) \\ \tilde{\theta} \sim U(r^{j+1}, r^j) & \text{if } \hat{a} \in [r^j, r^{j-1}) \text{ for } j = 2,3,4,\ldots \\ \{\theta = 0\} & \text{if } \hat{a} = 0 \end{cases}$$

Inspection of this belief function shows that investors' expectation of firm type weakly decrease as the announcement decreases. If the highest equilibrium announcement, $\hat{a} = 1$, is observed, then investors believe the firm's type is uniform on the highest interval, $(r, 1]$. If an out-of-equilibrium announcement $\hat{a}_o$ is observed such that $r < \hat{a}_o < 1$, investors believe the firm's type is uniform on the second highest interval, $\left(r^2, r\right]$. If the second highest equilibrium announcement, $\hat{a} = r$, is observed, investors believe the firm's type is uniform on the second highest interval, $\left(r^2, r\right]$. If an out-of-equilibrium announcement $\hat{a}_o$ is observed such that $\hat{a}_o$ is strictly between r^{j+1} and r^j, then investors believe the firm's type is uniform on the next lower interval $\left(r^{j+2}, r^{j+1}\right]$.

The investor belief function described above satisfies (A-6) at the end of Chapter 3. Assumption (A-6) is a weak monotonicity requirement that investors' posterior expectation of firm type be weakly increasing as the announcement increases.

Table 3 on the following page compares the investors' posterior beliefs and responses for various announcements.

Table 3. Investors' beliefs and response in partial pooling case.

Partition of feasible types: $\{\ldots,(r^{j+2},r^{j+1}],(r^{j+1},r^{j}],(r^{j},r^{j-1}],\ldots,(r^{2},r],(r,1]\}$

Equilibrium announcements: $\hat{A}^{e}=\{r^{j}\,|\,j=0,1,2,3,\ldots\}=\{\ldots,r^{j+1},r^{j},r^{j-1},\ldots,r^{2},r,1\}$

Out-of-equilibrium announcement set: $\hat{A}^{o}=\{\hat{a}\,|\,\hat{a}\in[0,1]\text{ and }\hat{a}\notin\hat{A}^{e}\}$

Observed announcement, $\hat{a}$	Beliefs about type, $\mu(\tilde{\theta}\mid\hat{a})$	Expectation of type, $E[\tilde{\theta}\mid\mu(\tilde{\theta}\mid\hat{a})]$	Investors' belief about allocation, $a^{rpp}(\theta)$	Investors' valuation response at time 1, $V_1^{rpp}(\hat{a})$
Equilibrium announcements:				
1	$U(r,1)$	$\frac{r+1}{2}$	$\frac{\theta+2\hat{k}}{1+2\hat{k}}$	$\frac{\int_r^1 Z(\theta,1,a^{rpp}(\theta))\frac{1}{1-r}d\theta}{1+\beta_1+\beta_2}$
r	$U(r^2,r)$	$\frac{r^2+r}{2}$	$\frac{ka^*(\theta)+\hat{k}\hat{a}}{k+\hat{k}}$	$\frac{Z(\theta,\hat{a},a)}{1+\beta_1+\beta_2}$
r^j for j=2,3,...	$U(r^{j+1},r^j)$	$\frac{r^{j+1}+r^j}{2}$	$\frac{\theta+2\hat{k}r^j}{1+2\hat{k}}$	$\frac{Z(\theta,\hat{a},a)}{1+\beta_1+\beta_2}$
Out-of-equilibrium announcements:				
$\hat{a}\in(r^{j+1},r^j)$ for j=0,1,2,...	$U(r^{j+2},r^{j+1})$	$\frac{r^{j+2}+r^{j+1}}{2}$	$\frac{\theta+2\hat{k}\hat{a}}{1+2\hat{k}}$	$\frac{Z(\theta,\hat{a},a)}{1+\beta_1+\beta_2}$
$\hat{a}=0$	0	0	0	$\frac{Z(0,0,0)}{1+\beta_1+\beta_2}=\frac{0.375}{1+\beta_1+\beta_2}$

Proposition 5 shows that the proposed firm strategy and investor response is an equilibrium. Lemma 2 and Lemma 3 are useful in proving in Proposition 5.

Comment on Lemma 2: In this model the upper bound of each interval is the type $\theta = r^j$ for j=0,1,2,... . Indifference condition **(I)** specifies r must satisfy a particular functional relation with the exogenous parameters, β_1, β_2, and $\hat{k}$. If r satisfies **(I)**, then Lemma 2 shows these boundary types are indifferent between truthfully announcing $\hat{a}*(r^j)$ or mimicking the announcement of the next highest interval, $\hat{a}*(r^{j-1})$. This characteristic is similar to the boundary indifference conditions that characterize the partition equilibria in Crawford -Sobel [p. 1437] and Newman-Sansing.

Lemma 2. Given a parameter value r such that $0 < r < 1$ and r is a solution to

$$\Omega(r) = \beta_1(1+r)(1+r+r^2) + 6\hat{k}\{\beta_1 r(1+r) - \beta_2(1+\beta_1+\beta_2)(1-r)\} = 0, \quad \textbf{(I)}$$

and firm's allocation $a^{\text{rpp}}(\theta) = \dfrac{\theta + 2\hat{k}r^{j}}{1+2\hat{k}}$ for $\theta \in (r^{j+1}, r^{j}]$,

and $V_1^{\text{rpp}}(\hat{a}) = \dfrac{1}{1+\beta_1+\beta_2} \displaystyle\int_{r^j}^{r^{j-1}} Z(\theta, \hat{a}, a^{\text{rpp}}(\theta)) \frac{1}{r^j - r^{j+1}} d\theta$ for $\hat{a} \in [r^j, r^{j-1})$;

then any boundary type $\theta = r^j$ where j=1,2,..., is indifferent between announcing $\hat{a}*(r^j)$ or $\hat{a}*(r^{j-1})$.

Discussed in the main text at page 91.

Proof: Define the payoff to boundary type $\theta = r^j$ from announcing $\hat{a}*(r^j)$ as

$$W^{\text{truth}}(r^j) \equiv W(r^j,\ \hat{a}*(r^j),\ a*(r^j),\ V_1(\hat{a}*(r^j))).$$

Define the boundary type's payoff from announcing $\hat{a}*(r^{j-1})$ as

$$W^{\text{mimic}}(r^j) \equiv W(r^j,\ \hat{a}*(r^{j-1}),\ a^c(r^j, \hat{a}*(r^{j-1})),\ V_1(\hat{a}*(r^{j-1}))).$$

Substituting and simplifying, $W^{\text{truth}}(r^j) - W^{\text{mimic}}(r^j) = \dfrac{1}{\tau_6}(r-1)\, r^{2j-2}\, \Omega(r)$

where $\Omega(r)$ is defined above and $\tau_6 = 6(1+\beta_2)(1+\beta_1+\beta_2)(1+2\hat{k}) > 0$

By **(I)** the value of r satisfies $\Omega(r) = 0$, which implies $W^{\text{truth}}(r^j) - W^{\text{mimic}}(r^j)$=0.

Thus the boundary type is indifferent between announcing $\hat{a}*(r^j)$ or $\hat{a}*(r^{j-1})$. ∎

Comment on Lemma 3: Condition **(I)** assumed there exists an r such that $0 < r < 1$ and $\Omega(r) = 0$. Lemma 3 shows that under certain parameter conditions there always exists a real number r that satisfies **(I)**.

Lemma 3. If $\beta_1 > 0$, $\beta_2 > 0$, $k = \frac{1}{2}$, $\hat{k} > \frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$, then there exists a solution r such that satisfies **(I)**.

Discussed in the main text at page 91.

Proof: Observe $\Omega(r)$ is a continuous function for $r \in [0,1]$,

$\Omega(0) = \beta_1 - 6\beta_2(1+\beta_1+\beta_2)\hat{k} < 0$, and $\Omega(1) = 6\beta_1(1+2\hat{k}) > 0$.

By the Intermediate Value Theorem, there exists at least one r such that $0 < r < 1$ and $\Omega(r) = 0$. Thus, there exists at least one r that satisfies assumption **(I)**. ■

Proposition 5. If the firm privately observes its type, and the penalty factor $\hat{k}$ is a sufficiently large finite value, such that $\frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)} < \hat{k} < +\infty$, then a partial pooling equilibrium occurs. This partial pooling equilibrium is characterized by:

(i) Almost all firms lie.

(ii) Almost all firms distort.

(iii) There are an infinite number of intervals.

Discussed in text at page 88.

Proof:

Assume the following exogenous parameter values: $\beta_1 > 0$, $\beta_2 > 0$, $k = \frac{1}{2}$, and $\hat{k} > \frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$. By Lemma 3 there exists an r such that $0 < r < 1$ and

$$\Omega(r) = \beta_1(1+r)(1+r+r^2) + 6\hat{k}\{\beta_1 r(1+r) - \beta_2(1+\beta_1+\beta_2)(1-r)\} = 0 .$$

Assume investors' posterior beliefs about type are

$$\mu(\tilde{\theta}|\hat{a}) = \begin{cases} \tilde{\theta} \sim U(r,1) & \text{if } \hat{a} = 1 \\ \tilde{\theta} \sim U(r^2,r) & \text{if } \hat{a} \in [r,1) \\ \tilde{\theta} \sim U(r^{j+1},r^j) & \text{if } \hat{a} \in [r^j,r^{j-1}) \\ \{\theta = 0\} & \text{if } \hat{a} = 0 \end{cases}$$

This belief function implies that if investors observe an out-of-equilibrium announcement is observed such that $r^j < \hat{a} < r^{j+1}$, then investors believe the type is uniform on the jth interval, $\theta \in (r^{j+1}, r^j]$. If an out-of-equilibrium announcement is observed such that $r^{j-1} < \hat{a} < r^{j-2}$, then investors believe the type is uniform on the (j-1) interval, $\theta \in \left(r^{j}, r^{j-1}\right]$.

Assume the investors use the observed announcement $\hat{a}$ and the inferred type θ to infer that the firm's allocation is $a^{rpp}(\theta) = \dfrac{\theta + 2\hat{k}r^{j}}{1+2\hat{k}}$ for $\theta \in \left(r^{j+1}, r^{j}\right]$.

Proposed equilibrium:

Firm's announcement: $\hat{a}^{rpp}(\theta) = \hat{a}*\left(r^{j}\right) = r^{j}$ for $\theta \in \left(r^{j+1}, r^{j}\right]$ and j=0,1,2,...

Set of announcements observed in equilibrium: $\hat{A}^{e} \equiv \left\{..., r^{j+1}, r^{j}, r^{j-1}, ...r^{2},\ r,\ 1\right\}$

Firm's allocation: $a^{rpp}(\theta) = \dfrac{\theta + 2\hat{k}r^{j}}{1+2\hat{k}}$ for $\theta \in \left(r^{j+1}, r^{j}\right]$

Investors' response: $V_{1}^{rpp}(\hat{a}) = \dfrac{1}{1+\beta_{1}+\beta_{2}}\displaystyle\int_{0}^{1} Z\left(\theta, \hat{a}, a^{rpp}(\theta)\right)\, d\mu\left(\tilde{\theta}\middle|\hat{a}\right)$

Outline of Proof: The first four steps of the proof show that the firm strategy and investors' response proposed above is an equilibrium. Steps 5 through 7 show this equilibrium has the characteristics labeled (i) thorough (iii) in Proposition 5.

Consider a type θ in the jth interval, such that $r^{j+1} < \theta \leq r^{j}$. The proposed equilibrium announcement and allocation pair is $\left\{r^{j},\ a^{rpp}(\theta)\right\}$. Consider a deviation to another pair, $\left\{\hat{a}_{o}, a_{o}\right\}$, such that $\hat{a}_{o} \neq r^{j}$. The proof shows that the payoff to the firm from the equilibrium strategy pair, $\{r^j,\ a^{rpp}(\theta)\}$, dominates the payoff from the out-of-equilibrium pair, $\{\hat{a}_o, a_o\}$.

Since there are an infinite number of feasible allocations, $a \in [0,1]$, and announcements, $\hat{a} \in [0,1]$, there are an infinite number of out-of-equilibrium pairs that could be tested. Step 1 of the proof identifies the allocation, $a^c(\theta, \hat{a})$, that maximizes the firm payoff given a particular announcement $\hat{a}$. Step 2 shows firms do not deviate to an out-of-equilibrium announcement, $\hat{a}_o \notin \hat{A}^e$. If a firm is in the *j*th interval, then the proposed equilibrium announcement is r^j. Step 3 shows that firms in the *j*th interval do not deviate to an announcement of an adjacent interval. Step 4 verifies the investors' response to the firm's announcement strategy is rational.

Step 1. Show that if a firm chooses announcement $\hat{a} = r^j$, then its optimal allocation is $a^{rpp}(\theta) = \dfrac{\theta + 2\hat{k}r^j}{1+2\hat{k}}$.

The firm's decision problem is $\underset{a}{\text{Max}}\ W\left(\theta, \hat{a} = r^j, a, V_1^{rpp}\left(\hat{a} = r^j\right)\right)$.

From Lemma 1, $W\left(\theta, r^j, a, V_1^{rpp}\left(r^j\right)\right) = \dfrac{\beta_1}{1+\beta_2} V_1^{rpp}\left(r^j\right) + \dfrac{\beta_2}{1+\beta_2} Z\left(\theta, r^j, a\right)$

Given assumption (A-4) in section 3.3 and assumption (A-7) in section 4.2,

$$Z\left(\theta, r^j, a\right) = \theta a + \frac{1}{2}(1-a) - \frac{1}{2}\left(a - \frac{1}{2}\right)^2 - \hat{k}\left(r^j - a\right)^2$$

The first-order condition is solved as follows.

$$\frac{\partial W\left(\theta, a, r^j, V_1^{rpp}(r^j)\right)}{\partial a} = \theta - a - 2\hat{k}\left(a - r^j\right) = 0$$

Solving the first-order condition for a yields $a = \dfrac{\theta + 2\hat{k}r^j}{1 + 2\hat{k}}$.

The second-order sufficient condition is satisfied,

$$\frac{\partial^2 W\left(\theta, a, \hat{a}, V_1^{rpp}(\hat{a})\right)}{\partial a^2} = -1 - 2\hat{k} < 0$$

Thus, the choice of a that will maximize the firm's compensation when the firm announces $\hat{a} = r^j$ is given by $a^{rpp}(\theta) = \dfrac{\theta + 2\hat{k}r^j}{1 + 2\hat{k}}$.

Step 2. Show that no type deviates to an announcement not in the equilibrium announcement set, $\hat{A}^e$.

Consider a deviation to an announcement $\hat{a}_o$, such that $r^{j+1} < \hat{a}_o < r^j$.

If the firm deviates to $\hat{a}_o$, then its optimal allocation is $a^c(\theta, \hat{a}_o) = \dfrac{\theta + 2\hat{k}\hat{a}_o}{1 + 2\hat{k}}$ as shown in Step 1. I compare the payoff from the in-equilibrium strategy pair, $\{r^j,\ a^{rpp}(\theta)\}$, to the payoff from the pair, $\{\hat{a}_o, a^c(\theta, \hat{a}_o)\}$.

If the investors observe the in-equilibrium announcement, r^j, then they believe the firm type is in the jth interval, and their response is

$$V_1^{rpp}(r^j) = \frac{1}{1 + \beta_1 + \beta_2} \int_{r^{j+1}}^{r^j} Z\left(\theta, r^j, a^{rpp}(\theta)\right) \frac{1}{r^j - r^{j+1}} d\theta$$

If the investors observe $\hat{a}_o$, such that $r^{j+1} < \hat{a}_o < r^j$, then they believe the firm type is in the next lower interval, $\left(r^{j+2}, r^{j+1}\right]$, and their response is

$$V_1^{rpp}(\hat{a}_o) = \frac{1}{1+\beta_1+\beta_2}\int_{r^{j+2}}^{r^{j+1}} Z(\theta,\hat{a}_o,a^c(\theta,\hat{a}_o))\frac{1}{r^{j+1}-r^{j+2}}d\theta.$$

I want to show the firm's payoff from the in-equilibrium announcement-allocation strategy $\{r^j,\ a^{rpp}(\theta)\}$ dominates the payoff from the out-of-equilibrium strategy pair, $\{\hat{a}_o, a^c(\theta,\hat{a}_o)\}$ for $\theta \in \left(r^{j+1}, r^j\right]$ and $\hat{a}_o \in \left(r^{j+1}, r^j\right)$.

Define $\Delta(\theta,\hat{a}_o)$ as the difference in payoffs.

$$\Delta(\theta,\hat{a}_o) \equiv W\left(\theta,\ r^j,\ a^{rpp}(\theta),\ V_1^{rpp}(r^j)\right) - W\left(\theta,\ \hat{a}_o,\ a^c(\theta,\hat{a}_o),\ V_1^{rpp}(\hat{a}_o)\right)$$

If $\Delta(\theta,\hat{a}_o) > 0$, then the type θ firm strictly prefers the in-equilibrium pair $\{r^j,\ a^{rpp}(\theta)\}$ over the out-of-equilibrium pair, $\{\hat{a}_o, a^c(\theta,\hat{a}_o)\}$.

Substituting in results (1) and (2) from Lemma 1 and evaluating yields

$$\Delta(\theta,\hat{a}_o) = \frac{\beta_1}{1+\beta_2}\left\{V_1^{rpp}(r^j) - V_1^{rpp}(\hat{a}_o)\right\} -$$

$$\frac{\beta_2}{1+\beta_2}\left\{Z(\theta,\hat{a}_o,a^c(\theta,\hat{a}_o)) - Z(\theta,r^j,a^c(\theta,r^j))\right\}$$

$$= \frac{\beta_1}{1+\beta_2}\frac{r^{2j}(1+r)(1-r^3)+6\hat{k}\left\{(r^j-\hat{a}_o)^2+r^{2j}(1-r^2)+(r^j-\hat{a}_o)r^j(r^2+r-2)\right\}}{6(1+\beta_1+\beta_2)(1+2\hat{k})}$$

$$-\frac{\beta_2}{1+\beta_2}\frac{(r^j-\hat{a}_o)\hat{k}\left(2(r^j-\theta)-(r^j-\hat{a}_o)\right)}{(1+2\hat{k})}$$

Differentiating with respect to θ shows $\Delta(\theta, \hat{a}_o)$ is an increasing function of θ.

$$\frac{\partial\Delta(\theta, \hat{a}_o)}{\partial\theta} = \frac{\beta_2}{1+\beta_2}\frac{2\hat{k}(r^j - \hat{a}_o)}{(1+2\hat{k})} > 0 \text{ because } \hat{a}_o < r^j$$

Thus, $\Delta(\theta, \hat{a}_o) > \Delta(r^{j+1}, \hat{a}_o)$ for $\theta > r^{j+1}$. This implies that for types in the *j*th interval, $\theta \in (r^{j+1}, r^j]$, the advantage of the equilibrium strategy payoff over the out-of-equilibrium is least for types near the infimum, $\theta = r^{j+1}$. In other words, the type with the greatest incentive to deviate is the lowest type in the interval.

Evaluating $\Delta(\theta, \hat{a}_o)$ for $\theta = r^{j+1}$ and simplifying yields

$$\Delta(r^{j+1}, \hat{a}_o) = \frac{\lambda_0 + \lambda_1(r^j - \hat{a}_o) + \lambda_2(r^j - \hat{a}_o)^2}{6(1+\beta_2)(1+\beta_1+\beta_2)(1+2\hat{k})}$$

where $\lambda_0 \equiv \beta_1 r^{2j}(1-r)(1+r)\{1+6\hat{k}+r(1+r)\} > 0$

$\lambda_1 \equiv 6\hat{k}\, r^j(r-1)\{2(\beta_1+\beta_2)(1+\beta_2) + \beta_1 r\} < 0$ because $(r-1)<0$

$\lambda_2 \equiv 6\hat{k}\,(\beta_1+\beta_2)(1+\beta_2) > 0$

As $\hat{a}_o$ approaches r^j, and $\Delta(r^{j+1}, r^j) = \dfrac{\lambda_0}{6(1+\beta_1+\beta_2)(1+2\hat{k})} > 0$.

Differentiating $\Delta(r^{j+1}, \hat{a}_o)$ with respect to $\hat{a}_o$ shows that $\Delta(r^{j+1}, \hat{a}_o)$ is an increasing function for values of $\hat{a}_o > r^{j+1}$.

$$\frac{\partial\Delta(r^{j+1}, \hat{a}_o)}{\partial\hat{a}_o} = \frac{2\hat{k}(\beta_1+\beta_2)(1+\beta_2)(\hat{a}_o - r^{j+1}) + \hat{k}r^j(1-r)\beta_1 r}{6(1+\beta_2)(1+\beta_1+\beta_2)(1+2\hat{k})} > 0 \text{ for } \hat{a}_o > r^{j+1}$$

This implies $\Delta(r^{j+1}, \hat{a}_o) > \Delta(r^{j+1}, r^{j+1})$ for $\hat{a}_o > r^{j+1}$.

Evaluating $\Delta\left(r^{j+1}, \hat{a}_o\right)$ at $\hat{a}_o = r^{j+1}$ and simplifying yields

$$\Delta\left(r^{j+1}, r^{j+1}\right) = \frac{r^{2j}\,(1-r)\,\Omega(r)}{6\left(1+\beta_1+\beta_2\right)\left(1+2\hat{k}\right)} = 0, \text{ because } \Omega(r) = 0.$$

This last result implies the type $\theta = r^{j+1}$ is indifferent between announcing, r^{j+1} or r^{j} . Recall that Lemma 2 showed that types at the interval boundaries are indifferent between the announcements of adjacent intervals.

Combining the preceding results yields, $\Delta(\theta, \hat{a}_o) > \Delta\left(r^{j+1}, \hat{a}_o\right) > \Delta\left(r^{j+1}, r^{j+1}\right) = 0$ for all $\theta \in \left(r^{j+1}, r^{j}\right]$, and all $\hat{a}_o \in \left(r^{j+1}, r^{j}\right)$.

This implies $W\left(\theta,\ r^{j},\ a^{rpp}(\theta),\ V_1^{rpp}\left(r^{j}\right)\right) > W\left(\theta,\ \hat{a}_o,\ a^{c}\left(\theta, \hat{a}_o\right),\ V_1^{rpp}\left(\hat{a}_o\right)\right)$.

Thus, Step 2 shows the payoff from the proposed equilibrium strategy pair, $\left\{r^{j},\ a^{rpp}(\theta)\right\}$, dominates the payoff from the pair, $\left\{\hat{a}_o, a^{c}\left(\theta, \hat{a}_o\right)\right\}$ for all types in the *j*th interval considering a deviation to an out-of-equilibrium announcement $\hat{a}_o \in \left(r^{j+1}, r^{j}\right)$.

Similarly, it can be shown types in the *j*th interval prefer to announce r^{j} rather than deviating to other out-of-equilibrium announcements, such as $\hat{a}_o \in \left(r^{j+2}, r^{j+1}\right)$, or $\hat{a}_o \in \left(r^{j}, r^{j-1}\right)$.

Step 3. Show the firm does not deviate to the next higher interval's announcement, $\hat{a} = r^{j-1}$.

If the investors observe the announcement, $\hat{a} = r^{j-1}$, they believe the firm's type is uniform on the interval $\left(r^j, r^{j-1}\right]$. If the firm announces $\hat{a} = r^{j-1}$, then by Step 1 the firm's best allocation is $a^c\left(\theta, r^{j-1}\right) = \dfrac{\theta + 2\hat{k}\, r^{j-1}}{1+2\hat{k}}$.

I want to show that the firm's payoff from the in-equilibrium announcement-allocation strategy $\{r^j,\ a^{rpp}(\theta)\}$ dominates the payoff from the out-of-equilibrium strategy pair, $\{r^{j-1}, a^c(\theta, r^{j-1})\}$ for $\theta \in \left(r^{j+1}, r^j\right]$.

Define $\Delta(\theta)$ as the difference in payoffs for $\theta \in \left(r^{j+1}, r^j\right]$.

$$\Delta(\theta) \equiv W\left(\theta,\ r^j,\ a^{rpp}(\theta),\ V_1^{rpp}(r^j)\right) - W\left(\theta,\ r^{j-1},\ a^c(\theta, r^{j-1}),\ V_1^{rpp}(r^{j-1})\right)$$

If $\Delta(\theta) > 0$, then the type θ firm strictly prefers the in-equilibrium pair $\{r^j,\ a^{rpp}(\theta)\}$ over the out-of-equilibrium pair, $\{r^{j-1}, a^c(\theta, r^{j-1})\}$.

Substituting in results (1) and (2) from Lemma 1 and rearranging yields

$$\Delta(\theta) = \frac{\beta_1}{1+\beta_2}\left\{V_1^{rpp}(r^j) - V_1^{rpp}(r^{j-1})\right\} +$$

$$\frac{\beta_2}{1+\beta_2}\left\{Z\left(\theta, r^j, a^c(\theta, r^j)\right) - Z\left(\theta, r^{j-1}, a^c(\theta, r^{j-1})\right)\right\}$$

The first term in the above statement is the change in payoff at time 1 from announcing r^{j-1} rather than r^j; and the second term, the change in penalty at time 2. Evaluating and simplifying yields,

$$\Delta(\theta)=\frac{\beta_1}{(1+\beta_2)}\frac{r^{2j-2}(r^2-1)(1+r+6\hat{k}r+r^2)}{6(1+\beta_1+\beta_2)(1+2\hat{k})}+\frac{\beta_2}{(1+\beta_2)}\frac{\hat{k}(r-1)r^{j-1}(r^{j-1}+r^j-2\theta)}{(1+2\hat{k})}$$

$$=\frac{r^{j-1}(r-1)\{c+6\hat{k}(\beta_1 r^{j+1}(1+r)-\beta_2(1+\beta_1+\beta_2)r(r^{j-1}+r^j-2\theta))\}}{6(1+\beta_2)(1+\beta_1+\beta_2)(1+2\hat{k})}$$

where $c\equiv\beta_1 r^j(1+r)(1+r+r^2)$

Differentiating, $\frac{\partial\Delta(\theta)}{\partial\theta}=\frac{r^{j-1}(r-1)\{6\hat{k}(-\beta_2(1+\beta_1+\beta_2)r(-2))\}}{6(1+\beta_2)(1+\beta_1+\beta_2)(1+2\hat{k})}<0$ because $(r-1)<0$. Therefore, $\Delta(\theta)$ decreases as θ increases, and $\Delta(\theta)>\Delta(r^j)$ for all $\theta<r^j$.

$\Delta(r^j)$ simplifies to $\frac{r^{2j-2}(1-r)\Omega(r)}{6(1+\beta_2)(1+\beta_1+\beta_2)(1+2\hat{k})}=0$ because $\Omega(r)=0$. This implies the type at the upper boundary of the *j*th interval, $\theta=r^j$, is indifferent between announcing r^j and the next higher in-equilibrium announcement, r^{j-1}.

Finally, observe $\Delta(\theta)>\Delta(r^j)=0$ for all $\theta<r^j$. Thus, all types strictly within the *j*th interval strictly prefer the in-equilibrium strategy $\{r^j,\ a^{rpp}(\theta)\}$ over the out-of-equilibrium pair, $\{r^{j-1}, a^c(\theta, r^{j-1})\}$.

Similarly, it can be shown types in the *j*th interval prefer to announce r^j rather than deviating to the next lower announcement, r^{j+1}.

Step 4. Given $a^{rpp}(\theta)$, and $\hat{a}^{rpp}(\theta)$, show the investors' best-response is $V_1^{rpp}(\hat{a}^{rpp}(\theta))$.

Suppose $\theta \in (r^{j+1},\ r^{j}]$, then the observed announcement is $\hat{a}^{rpp}(\theta) = r^{j}$.

After observing the announcement $\hat{a} = r^{j}$, the investors' belief function, $\mu(\tilde{\theta}|\hat{a} = r^{j})$ is evaluated, and investors believe $\tilde{\theta} \sim U(r^{j+1}, r^{j})$. $V_1^{rpp}(\hat{a}^{rpp}(\theta))$ is the expectation taken over this interval.

$$V_1^{rpp}(\hat{a}^{rpp}(\theta)) = \frac{1}{1+\beta_1+\beta_2}\int_{r^{j+1}}^{r^{j}} Z(\theta, r^{j}, a^{rpp}(\theta))\frac{1}{r^{j}-r^{j+1}}\,d\theta$$

The investors' belief about the firm's allocation is consistent with the firm's equilibrium allocation rule. If an out-of-equilibrium announcement, $\hat{a} \notin \hat{A}^{e}$ is observed, then the investors' response is sequentially rational given their beliefs.

Step 5. Show that in this equilibrium nearly all firms lie.

Lying is defined as the difference between the announcement and the allocation. In this equilibrium, the amount of lying for types in interval j is

$\hat{a}^{rpp}(\theta) - a^{rpp}(\theta) = \dfrac{r^{j}-\theta}{1+2\hat{k}}$ for $\theta \in (r^{j+1}, r^{j}]$. For types strictly within interval *j*, the amount of lying is strictly positive. In other words all types make an announcement that overstates the allocation. The boundary type, $\theta = r^{j}$, does not lie, because $\hat{a}^{rpp}(r^{j}) - a^{rpp}(r^{j}) = r^{j} - r^{j} = 0$.

Step 6. Show that in this equilibrium nearly all firms distort.

Distortion is defined as the difference between the allocation chosen and the cash-maximizing allocation, $a^*(\theta)$. In this equilibrium, the amount of distortion for types in interval j is $a^{rpp}(\theta) - a^*(\theta) = \dfrac{2\hat{k}(r^j - \theta)}{1+2\hat{k}}$ for $\theta \in (r^{j+1}, r^j]$. For types strictly within interval j, the amount of distortion is strictly positive. In other words, all types make an allocation greater than the cash-maximizing allocation. The boundary type, $\theta = r^j$, does not distort, because $a^{rpp}(r^j) - a^*(r^j) = r^j - r^j = 0$.

Step 7. Show that this equilibrium is characterized by an infinite number of intervals.

To prove that the number of intervals is infinite, I show that assuming a finite number of intervals leads to a contradiction.

Suppose there are N intervals with supremum labeled j=0,1,...,N-1. The lowest interval is $[0, r^{N-1}]$, and all $\theta \in [0, r^{N-1}]$ should announce $\hat{a} = r^{N-1}$. Thus, $\hat{a} = 0$ should be an out-of-equilibrium announcement. Assume that if $\hat{a} = 0$ is observed, then the investors assume the firm is lowest type, θ=0.

Define the investors' response in the proposed equilibrium with N intervals as $V_1^{rN}(\hat{a}) = \dfrac{1}{1+\beta_1+\beta_2}\displaystyle\int_0^1 Z(\theta, \hat{a}, a^{rpp}(\theta))\, d\mu(\tilde{\theta}|\hat{a})$

$$\text{where } \mu(\tilde{\theta}|\hat{a}) = \begin{cases} \tilde{\theta} \sim U(r,1) & \text{if } \hat{a} = 1 \\ \tilde{\theta} \sim U(r^2,r) & \text{if } \hat{a} \in [r,1) \\ \tilde{\theta} \sim U(r^{j+1},r^j) & \text{if } \hat{a} \in [r^j, r^{j-1}) \text{ for } j = 1,\dots,N-1 \\ \{\theta = 0\} & \text{if } \hat{a} \in [0, r^{N-1}) \end{cases}$$

In this proposed equilibrium with N intervals the lowest type should prefer to make the announcement of interval N rather than deviate to the announcement $\hat{a} = 0$. Thus for type $\theta = 0$, the payoff from the strategy $\{\hat{a} = 0, a = 0\}$ should be strictly less than the payoff from the strategy $\left\{\hat{a} = r^{N-1}, a = \frac{2\hat{k}r^{N-1}}{1+2\hat{k}}\right\}$.

This implies: $$W(0,\ 0,\ 0,\ V_1^{rN}(0)) < W\left(0,\ r^{N-1},\ \frac{2\hat{k}r^{N-1}}{1+2\hat{k}},\ V_1^{rN}(r^{N-1})\right)$$

and $$W(0,\ 0,\ 0,\ V_1^{rN}(0)) - W\left(0,\ r^{N-1},\ \frac{2\hat{k}r^{N-1}}{1+2\hat{k}},\ V_1^{rN}(r^{N-1})\right) < 0.$$

Evaluating and simplifying yields the following result.

$$\frac{r^{2N-2}\left\{6\beta_2(1+\beta_1+\beta_2)\hat{k} - \beta_1\right\}}{6(1+\beta_2)(1+\beta_1+\beta_2)(1+2\hat{k})} < 0$$

Given the parameter restrictions this implies $\left\{6\beta_2(1+\beta_1+\beta_2)\hat{k} - \beta_1\right\} < 0$ and

$\hat{k} < \frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$. However, this contradicts the assumption in Proposition 5

that $\hat{k}$ is sufficiently large that $\hat{k} > \frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$.

This implies the lowest type does prefer the strategy $\{\hat{a}=0, a=0\}$ over the strategy $\left\{\hat{a}=r^{N-1}, a=\frac{2\hat{k}r^{N-1}}{1+2\hat{k}}\right\}$.

The value of $\hat{k}$ is critical in this model. For $\hat{k}<\frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$ the lowest type strictly prefers to mimic the highest type and the equilibrium has a single pool with $\hat{a}=1$. When $\hat{k}>\frac{\beta_1}{6\beta_2(1+\beta_1+\beta_2)}$ the cost of mimicking is so large that the lowest type strictly prefers to announce $\hat{a}=0$; and there is a partial pooling equilibrium with an infinite number of intervals. ■

Proposition 6. If the firm privately observes its type and the penalty factor $\hat{k}$ is infinitely large, then a partial pooling equilibrium occurs. This partial pooling equilibrium is characterized by:

(i) No firm lies.

(ii) Almost all firms distort.

(iii) There are an infinite number of intervals.

Discussed in text at page 103.

Proof:
Assume the following exogenous parameter values:

$$\beta_1 > 0, \quad \beta_2 > 0, \quad k = \frac{1}{2}, \quad \hat{k} = +\infty,$$

r that satisfies $0 < r < 1$ and $\Gamma(r) = 0$

where $\Gamma(r) = -\beta_2(1+\beta_1+\beta_2) + \{\beta_1 + \beta_2(1+\beta_1+\beta_2)\}r + \beta_1 r^2$

Assume the investors' beliefs about type are

$$\mu(\tilde{\theta}|\hat{a}) = \begin{cases} \tilde{\theta} \sim U(r,1) & \text{if } \hat{a} = 1 \\ \tilde{\theta} \sim U(r^2,r) & \text{if } \hat{a} \in [r,1) \\ \tilde{\theta} \sim U(r^{j+1},r^j) & \text{if } \hat{a} \in [r^j,r^{j-1}) \\ \{\theta = 0\} & \text{if } \hat{a} = 0 \end{cases}$$

Assume the investors believe the firm's announcement is a truthful revelation about its allocation, $\hat{a} = a$.

Proposed equilibrium:

Equilibrium strategies given $\hat{k} = +\infty$ are denoted with the superscript $r\infty$.

Firm's announcement: $\hat{a}^{r\infty}(\theta) = r^j$ for $\theta \in (r^{j+1}, r^j]$ and j=0,1,2,...

Set of announcements observed in equilibrium: $\hat{A}^e \equiv \{\ldots, r^{j+1}, r^j, r^{j-1}, \ldots r^2,\ r,\ 1\}$

Firm's allocation equals their announcement: $a^{r\infty}(\theta) = \hat{a}^{r\infty}(\theta)$ for all $\theta \in [0,1]$

Investors' response: $V_1^{r\infty}(\hat{a}) = \frac{1}{1+\beta_1+\beta_2} \int_0^1 Z(\theta, \hat{a}, \hat{a})\ d\mu(\tilde{\theta}|\hat{a})$

Step 1. Show that lying is eliminated when $\hat{k} = +\infty$.

From Lemma 1 and assumption (A-7), the firm's payoff is

$$W(\theta, \hat{a}, a, V_1(\hat{a})) = \frac{\beta_1}{1+\beta_2} V_1(\hat{a}) + \frac{\beta_2}{1+\beta_2} Z(\theta, \hat{a}, a)$$

where $Z(\theta, \hat{a}, a) = \theta a + \frac{1}{2}(1-a) - \frac{1}{2}\left(a - \frac{1}{2}\right)^2 - \hat{k}(\hat{a}-a)^2$

When all parameters except $\hat{k}$ are fixed at arbitrary positive values and $\hat{k}$ approaches positive infinity, then the magnitude of the penalty exceeds all other terms.

$$\lim_{\hat{k}\to\infty} W(\theta, \hat{a}, a, V_1(\hat{a})) = \frac{\beta_2}{1+\beta_2}\left\{-\ \hat{k}(\hat{a}-a)^2\right\}$$

The firm avoids the penalty by choosing an annou.icement equal to the allocation, $\hat{a} = a$, which is defined as no lying.

Step 2. When $\hat{k}$ approaches infinity and $\Gamma(r) = 0$, show that the boundary types, $\theta = r^j$ for *j*=1,2,3,... are indifferent between interval *j* (announcing $\hat{a} = r^j$) and the adjacent interval *j*-1 (announcing $\hat{a} = r^{j-1}$).

Lemma 2 shows the indifference condition (I) such that the boundary types are indifferent. Condition (I) requires an r that satisfies $0 < r < 1$ and

$$\Omega(r) = \beta_1(1+r)(1+r+r^2) + 6\hat{k}\{\beta_1 r(1+r) - \beta_2(1+\beta_1+\beta_2)(1-r)\} = 0$$

Collecting terms of $\hat{k}$ yields

$$\Omega(r) = \beta_1(1+r)(1+r+r^2) + 6\hat{k}\Gamma(r) = 0$$

where $\Gamma(r) = -\beta_2(1+\beta_1+\beta_2) + \{\beta_1+\beta_2(1+\beta_1+\beta_2)\}r + \beta_1 r^2$

Taking the limit as $\hat{k}$ approaches infinity yields: $\lim_{\hat{k}\to\infty}\psi(r) = r^{j-1}6\hat{k}\Gamma(r) = 0$.

Given the assumed parameter values, $\Gamma(0) < 0$ and $\Gamma(1) = 2\beta_1 > 0$.

Since $\Gamma(r)$ is a continuous function, by the Intermediate Value Theorem, there exists at least one r such that r is a solution to $\Gamma(r) = 0$ and $0 < r < 1$.

Thus, given the exogenous parameters assumed above, $\hat{k} = +\infty$ and $\Gamma(r) = 0$, the boundary types are indifferent between the announcements of adjacent intervals.

Step 3. Show that all types other than the boundary types strictly prefer the proposed equilibrium rather than deviating to an out-of-equilibrium strategy. The details of this step are similar to Proposition 5 for the special case where $\hat{k} = +\infty$.

In Proposition 5, the firm's allocation was $a^{rpp}(\theta) = \dfrac{\theta + 2\hat{k}r^j}{1 + 2\hat{k}}$.

When $\hat{k} = +\infty$, $\lim_{\hat{k}\to\infty} a^{rpp}(\theta) = \lim_{\hat{k}\to\infty}\dfrac{\theta + 2\hat{k}r^j}{1 + 2\hat{k}} = r^j = a^{r\infty}(\theta)$.

The investors' beliefs in this proposition have the same functional form in Proposition 5. Proposition 5 showed that no type strictly prefers to deviate to an out-of-equilibrium strategy. Thus, this partial pooling is an equilibrium.

Step 4. Show that when the firm privately observes its type and $\hat{k} = +\infty$, partial pooling with no lying is the only equilibrium supported by reasonable beliefs.

Step 1 showed that lying is eliminated when $\hat{k} = +\infty$. The three possible categories of equilibria with no lying are full separation, pure pooling, and partial pooling.

In a full separation equilibrium with no lying each firm type has an announcement different from any other type. Corollary 2.1 implies that if there is a full separation equilibrium, the firm's best strategy is first-best. The first-best outcome is characterized by no lying and no distortion, $\{\hat{a}(\theta) = a^*(\theta)\}$. Corollary 3.2 showed the first-best outcome is not an equilibrium when $\hat{k} = +\infty$, because some firms distort.

In a pure pooling equilibrium with no lying, all firms make the same announcement and allocation, such as $\{\hat{a} = a = 1\}$ for all $\theta \in [0,1]$. If this pure pooling existed, then it would require severe out-of-equilibrium beliefs to prevent firms deviating to out-of-equilibrium announcements. One such set of severe beliefs is $\mu(\tilde{\theta}|\hat{a}) = \begin{cases} \tilde{\theta} \sim U(0,1) \text{ if } \hat{a} = 1 \\ \{\theta = 0\} \text{ if } \hat{a} < 1 \end{cases}$. These beliefs imply investors do not revise their beliefs unless only the worst announcement is observed. Such severe beliefs do not satisfy the monotonicity condition (A-6) specified at the end of chapter 3. (A-6) requires that investors' expectations of type are strictly increasing for some pair $\{\hat{a}', \hat{a}''\}$ such that $0 < \hat{a}' < \hat{a}'' < 1$. With the above severe beliefs, $\mathrm{E}[\tilde{\theta}|\hat{a}'] = \mathrm{E}[\tilde{\theta}|\hat{a}''] = 0$, for all $\{\hat{a}', \hat{a}''\}$ such that $0 < \hat{a}' < \hat{a}'' < 1$.

The previous steps of this proof showed partial pooling is an equilibrium. It is possible to verify that the beliefs specified at the beginning of this proof satisfy (A-6). If a pair of announcements is selected from within interval *j*, such that $r^{j+1} < \hat{a}' < \hat{a}'' < r^{j}$, then the investors' expectations are the same for either announcement, $\mathrm{E}\left[\tilde{\theta}\middle|\hat{a}'\right] = \mathrm{E}\left[\tilde{\theta}\middle|\hat{a}''\right] = \frac{r^{j+1} + r^{j}}{2}$. However, comparing announcements from adjacent intervals, shows investors' expectations are strictly increasing. For example, if $\hat{a}' \in \left(r^{j+1}, r^{j}\right]$ and $\hat{a}'' \in \left(r^{j}, r^{j-1}\right]$, then

$$\frac{r^{j+1} + r^{j}}{2} = \mathrm{E}\left[\tilde{\theta}\middle|\hat{a}'\right] < \mathrm{E}\left[\tilde{\theta}\middle|\hat{a}''\right] = \frac{r^{j} + r^{j-1}}{2}.$$

Step 5. Show that in this equilibrium nearly all firms distort their allocation away from the cash-maximizing allocation.

Distortion is defined as the difference between the allocation chosen and the cash-maximizing allocation, $a^{*}(\theta)$. When *k*=0.5, the cash-maximizing allocation is $a^{*}(\theta) = \theta$. In this equilibrium, the amount of distortion for types in interval *j* is $a^{r\infty}(\theta) - a^{*}(\theta) = r^{j} - \theta$ for $\theta \in \left(r^{j+1}, r^{j}\right]$. For types strictly within interval *j*, the chosen allocation is greater than the cash-maximizing allocation. The boundary type, $\theta = r^{j}$, does not distort, because $a^{r\infty}\left(r^{j}\right) - a^{*}\left(r^{j}\right) = r^{j} - r^{j} = 0$.

Step 6. Show that this partial pooling is characterized by an infinite number of intervals.

This step is essentially the same as the last step in the proof of Proposition 5. ■

BIBLIOGRAPHY

Allen, F., and G. R. Faulhaber. 1989. Signaling by underpricing in the IPO market. *Journal of Financial Economics* 23 (August): 303-323.

Alexander, J. C. 1991. Do the merits matter? A study of settlements in securities class actions. *Stanford Law Review* 43 (February): 497-598.

Bachar, J. 1989. Auditing quality, signaling, and underwriting contracts. *Contemporary Accounting Research* 6 (Fall): 216-241.

Bodner, S., and M. Kiss. 1988. Impairment of long-lived assets: an emerging controversial issue. *CPA Journal* 58 (July): 70-72.

Chang, O. H., and D. R. Nichols. 1991. Should impaired operating assets be written down? *Management Accounting* 73 (December): 29-32.

Cho, I., and D. M. Kreps. 1987. Signaling games and stable equilibria. *Quarterly Journal of Economics* 102 (May): 179-221.

Clark, H. G., and L. Lorensen. 1987. *Illustrations of accounting for the inability to fully recover the carrying amount of long-lived assets.* New York: AICPA.

Crawford, V. P., and J. Sobel. 1982. Strategic information transmission. *Econometrica* 50 (November): 1431-1451.

Darrough, M. A., and N. M. Stoughton. 1990. Financial disclosure policy in an entry game. *Journal of Accounting and Economics* 12 (January): 219-243.

Datar, S. M., G. A. Feltham, and J. S. Hughes. 1991. The role of audits and audit quality in valuing new issues. *Journal of Accounting and Economics* 14 (March): 3-50.

Dontoh, A. 1989. Voluntary disclosure. *Journal of Accounting, Auditing, and Finance* New Series 4 (Fall): 480-511.

Dye, R. A. 1985a. Disclosure of nonproprietary information. *Journal of Accounting Research* 23 (Spring): 123-145.

Dye, R. A. 1985b. Strategic accounting choice and the effects of alternative financial reporting requirements. *Journal of Accounting Research* 23 (Autumn): 544-574.

Dye, R. A. 1986. Proprietary and nonproprietary disclosure. *Journal of Business* 59 (April): 331-366.

Dye, R. A. 1988. Earnings management in an overlapping generations model. *Journal of Accounting Research* 26 (Autumn): 195-235.

Elliott, J. A., and W. H. Shaw. 1988. Write-offs as accounting procedures to manage perceptions. *Journal of Accounting Research* 26 (Supplement): 91-119.

Farrell, J. 1987. Cheap talk, coordination, and entry. *Rand Journal of Economics* 18 (Spring): 34-39.

Fedders, J. M., and L. G. Perry. 1984. Policing financial disclosure fraud: the SEC's top priority. *Journal of Accountancy* 158 (July): 58-64.

Feltham, G., and J-Z. Xie. 1992. Voluntary financial disclosure in an entry game with continua of types. *Contemporary Accounting Research* 9 (Fall): 46-80.

Fried, D., M. Schiff, and A. C. Sondhi. 1989. Impairments and writeoffs of long-lived assets. *Management Accounting* 71 (August): 48-50.

Green, J., and J. Laffont. 1990. Competition on many fronts: a Stackelberg signaling equilibrium. *Games and Economic Behavior* 2 (September): 247-272.

Grossman, S. J. 1976. On the efficiency of competitive stock markets where stock traders have diverse information. *Journal of Finance* 31 (May): 573-585.

Grossman, S. J. 1981. The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics* 24: 461-483.

Grossman, S. J., and O. Hart. 1980. Disclosure laws and takeover bids. *Journal of Finance* 35 (May): 323-334.

Harsanyi, J. C. 1968. Games with incomplete information played by Bayesian players, II: Bayesian equilibrium points. *Management Science* 14 (January): 320-334.

Heyman, J. A. 1989. SEC interprets management's discussion and analysis. *CPA Journal* 59 (December): 70-73.

Hooks, K. L., and J. E. Moon. 1991. A checklist for management discussion and analysis. *Journal of Accountancy* 172 (December): 95-101.

Hughes, P. J. 1985. Signalling by direct disclosure under asymmetric information. *Journal of Accounting and Economics* 8 (June): 119-142.

Jacobs, A. S. 1980. *The Impact of Rule 10b-5.* Revised edition. New York: Clark Boardman Company.

Jensen, M. C., and K. J. Murphy. 1990. Performance pay and top-management incentives. *Journal of Political Economy* 98 (April): 225-264.

Jovanovic, B. 1982. Truthful disclosure of information. *Bell Journal of Economics* 13 (Spring): 36-44.

Jung, W. 1989. Strategic choice of inventory accounting methods. *Contemporary Accounting Research* 6 (Fall): 1-25.

Jung, W., and Y. K. Kwon. 1988. Disclosure when the market is unsure of the information endowment of managers. *Journal of Accounting Research* 26 (Spring): 146-153.

Kambhu, J. 1988. Unilateral disclosure of private information by a regulated firm. *Journal of Economic Behavior and Organization* 10 (July): 57-82.

Kim, E. H., and J. D. Schatzberg. 1985. Voluntary corporate liquidations. *Journal of Financial Economics* 19 (December): 311-328.

Klein, A. 1986. The timing and substance of divestiture announcements: Individual, simultaneous, and cumulative effects. *Journal of Finance* 41 (July): 685-695.

Kreps, D., and R. Wilson. 1982. Reputation and imperfect information. *Journal of Economic Theory* 27 (August): 253-279.

Lanen, W. N., and Verrecchia, R. E. 1987. Operating decisions and the disclosure of management accounting information. *Journal of Accounting Research* 25 (Supplement): 165-193.

Lindahl, F. W., and W. E. Ricks. 1991. Market reactions to announcements of write-offs. Working paper, Duke University, Durham, North Carolina.

Loss, L. 1983. "Fraud" and civil liability under the federal securities laws. Washington, DC: Federal Judicial Center.

Lucas, R. E., Jr. 1978. Asset prices in an exchange economy. *Econometrica* 46 (November): 1429-1445.

Matthews, S. A., M. Okuno-Fujiwara, and A. Postlewaite. 1991. Refining cheap talk equilibria. *Journal of Economic Theory* 55 (December): 247-273.

Milgrom, P. R. 1981. Good news and bad news: representation theorems and applications. *Bell Journal of Economics* 12 (Autumn): 380-391.

Miller, M. H., and K. Rock. 1985. Dividend policy under asymmetric information. *Journal of Finance* 40 (September): 1031-1051.

Moses, J. M. 1992. Lawyer given to filing shareholder lawsuits comes under scrutiny. *Wall Street Journal* (October 28): A1 and A3.

Muth, J. F. 1961. Rational expectations and the theory of price movements. *Econometrica* 29: 315-335.

Myerson, R. 1979. Incentive compatibility and the bargaining problem. *Econometrica* 47 (January): 61-74.

Newman, P., and R. Sansing. 1993. Disclosure policies with multiple users. *Journal of Accounting Research* forthcoming.

Norman, J. R. 1989. Xerox rethinks itself -- and this could be the last time. *Business Week* 3091 (February 13): 90-93.

O'Brien, V. E. 1991. The class-action shakedown racket. *Wall Street Journal* (September 10): A20.

Rasmusen, E. 1989. *Games and Information: An Introduction to Game Theory.* Oxford: Basil Blackwell.

Reinganum, J. F., and L. L. Wilde. 1986. Equilibrium verification and reporting policies in a model of tax compliance. *International Economic Review* 27 (October): 739-760.

Sansing, R. 1990. Strategic auditing and strategic information transmission in reputation games. Unpublished dissertation, University of Texas at Austin.

Schuetze, W. 1987. Disclosure and the impairment question. *Journal of Accountancy* 164 (December): 26-32.

Securities and Exchange Commission. 1989. Financial Reporting Release No. 36: management's discussion and analysis of financial condition and results of operations. (May 18). Reprinted in L. Lorensen. 1991. *Illustrations of management's discussion and analysis of financial condition and results of operations* (Financial report survey 45). New York: AICPA, 99-115.

Skinner, D. J. 1992. Why firms voluntarily disclose bad news. Working paper, University of Michigan, Ann Arbor.

Statman, M., and J. F. Sepe. 1989. Project termination announcements and the market value of the firm. *Financial Management* 18 (Winter): 74-81.

Strong, J. S., and J. R. Meyer. 1987. Asset writedowns: Managerial incentives and security returns. *Journal of Finance* 42 (July): 643-63.

Teoh, S. H., and C. Y. Hwang. 1991. Nondisclosure and adverse disclosure as signals of firm value. *Review of Financial Studies* 4 (No. 2): 283-313.

Trueman, B. 1986. Why do managers voluntarily release earnings forecasts? *Journal of Accounting and Economics* 8 (March): 53-71.

Trueman, B. 1990. Theories of earnings announcement timing. *Journal of Accounting and Economics* 13 (October): 285-301.

Verrecchia, R. E. 1982. Information acquisition in a noisy rational expectations economy. *Econometrica* 50 (November): 1415-1430.

Verrecchia, R. E. 1983. Discretionary disclosure. *Journal of Accounting and Economics* 5 (December): 179-194.

Verrecchia, R. E. 1990. Information quality and discretionary disclosure. *Journal of Accounting and Economics* 12 (March): 365-380.

Xie, J. Z. 1991. Contract negotiation, incomplete contracting, and asymmetric information: essays in managerial accounting. Unpublished dissertation, University of British Columbia.

Wagenhofer, A. 1990. Voluntary disclosure with a strategic opponent. *Journal of Accounting and Economics* 12 (March): 341-363.

VITA

William Frederick Yancey was born in Boston, Massachusetts on August 30, 1956, the son of Joel Alexander Yancey and Marianne Yancey. After completing White Plains High School, White Plains, New York, in 1974, he entered Dartmouth College in Hanover, New Hampshire. While attending Dartmouth College, he had an internship with the USDA Forest Service in Washington, DC. In June 1978, he received a Bachelor of Arts from Dartmouth College, and entered the School of Forestry and Environmental Studies at Duke University, Durham, North Carolina. In December 1980, he received the Master of Forestry from Duke University. After several positions as an industrial forester, he became a Logging Business Management Instructor at Duluth Area Vocational Technical Institute, Duluth, Minnesota. In May 1983 he received the Bachelor of Accounting degree from the University of Minnesota, Duluth. In 1984 he received the Certified Public Accountant certificate from the Minnesota Board of Public Accountancy. From 1983 to 1987 he was employed as an accountant by Peat Marwick and several other accounting firms. In August 1987 he entered the Graduate School of The University of Texas at Austin. In August 1992 he became Assistant Professor of Accounting at the Neeley School of Business at Texas Christian University, Fort Worth, Texas.

Permanent address: 3801 Wren Avenue, Fort Worth, Texas 76133

This dissertation was typed by the author.